5^{th} Grade

Instructional Focus:

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

1. Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
2. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.
3. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real-world and mathematical problems.

Standard	Objective	Examples
Operations and Algebraic Thinking		
Write and interpret numerical expressions.		
5.OA.1. Use parentheses to construct numerical expressions, and evaluate numerical expressions with these symbols.	The student will understand and identify the difference between numerical expressions and equations. The student will explain the use of parentheses ( ) in an expression. The student will model the use parentheses ( ) in an expression	Background Info: Expressions are a series of numbers and symbols (+, -, x, ÷) without an equals sign. Equations result when two expressions are set equal to each other (2 + 3 = 4 + 1). 4(5 + 3) is an expression. When we compute 4(5 + 3) we are evaluating the expression. The expression equals 32. 4(5 + 3) = 32 is an equation. Example: • 15 - 7 – 2 = 10 → 15 - (7 – 2) = 10 • Compare 3 x 2 + 5 and 3 x (2 + 5) • Compare 15 – 6 + 7 and 15 – (6 + 7)
5.OA.2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognizing that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	Using written or verbal cues, TSW create on accurate expression using parentheses ( ), numbers, and symbols The student will Identify the correct expression from a set of possibilities that represent a multistep word problem.	Standard Example: Write an expression for the steps "double five and then add 26." Student (2 x 5) + 26 Describe how the expression 5(10 x 10) relates to 10 x 10. Student The expression 5(10 x 10) is 5 times larger than the expression 10 x 10 since I know that I that 5(10 x 10) means that I have 5 groups of (10 x 10).
Analyze patterns and relationships.		

<table>
<tr>
<td>5.OA.3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</td>
<td>The student will generate two numerical patterns using two given rules.

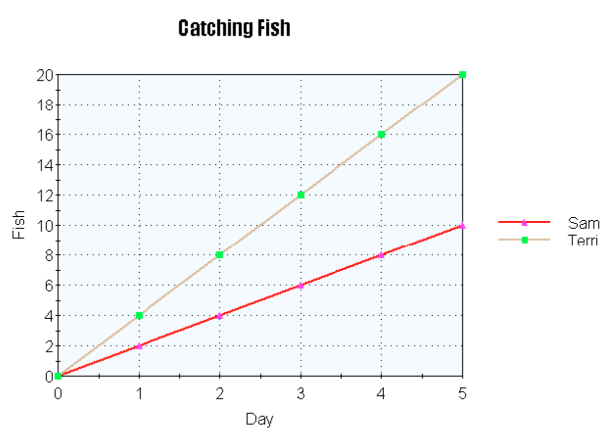
Given two rules with an apparent relationship, TSW identify relationship between the resulting sequences of terms in one sequence to the corresponding terms in the other sequence.

The student will form ordered pairs (consisting of corresponding terms from the two patterns), and graph the ordered pairs on a coordinate plane.</td>
<td>The graphs that are created should be line graphs to represent the pattern. This is a linear function which is why we get the straight lines. The Days are the independent variable, Fish are the dependent variables, and the constant rate is what the rule identifies in the table.
Make a chart (table) to represent the number of fish that Sam and Terri catch.
<table>
<tr><th>Days</th><th>Sam’s Total Number of Fish</th><th>Terri’s Total Number of Fish</th></tr>
<tr><td>0</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>2</td><td>4</td></tr>
<tr><td>2</td><td>4</td><td>8</td></tr>
<tr><td>3</td><td>6</td><td>12</td></tr>
<tr><td>4</td><td>8</td><td>16</td></tr>
<tr><td>5</td><td>10</td><td>20</td></tr>
</table>

Example:
Describe the pattern:
Since Terri catches 4 fish each day, and Sam catches 2 fish, the amount of Terri’s fish is always greater. Terri’s fish is also always twice as much as Sam’s fish. Today, both Sam and Terri have no fish. They both go fishing each day. Sam catches 2 fish each day. Terri catches 4 fish each day. How many fish do they have after each of the five days? Make a graph of the number of fish.

Plot the points on a coordinate plane and make a line graph, and then interpret the graph.

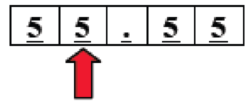
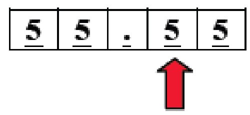
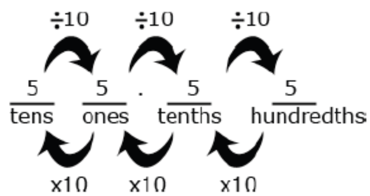
Student:
My graph shows that Terri always has more fish than Sam. Terri’s fish increases at a higher rate since she catches 4 fish every day. Sam only catches 2 fish every day, so his number of fish increases at a smaller rate than Terri.
Important to note as well that the lines become increasingly further apart. Identify apparent relationships between corresponding terms. Additional relationships: The two lines will never intersect; there will not be a day in which boys have the same total of fish, explain the relationship between the number of days that has passed and the number of fish a boy has (2n or 4n, n being the number of days).

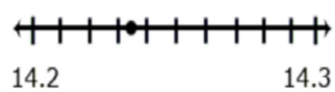


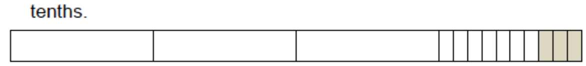
</td>
</tr>
</table>

Number and Operations in Base Ten

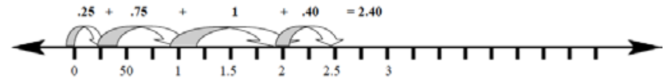
Understand the place value system.

5.NBT.1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	The student will explain the change in value of a number as it moves to the left or right on a place-value chart.	Before considering the relationship of decimal fractions, students express their understanding that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and 1/10 of what it represents in the place to its left. In the number 55.55, each digit is 5, but the value of the digits is different because of the placement.  The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the ones place is 1/10 of 50 and 10 times five tenths.  The 5 that the arrow points to is 1/10 of the 5 to the left and 10 times the 5 to the right. The 5 in the tenths place is 10 times five hundredths. 
5.NBT.2. Explain and extend the patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain and extend the patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	The student will recognize there is a relationship between the number of zeros in the factors and the number of zeros in the product. The student will model and explain the relationship between the number of zeros in the factors and the number of zeros in the product. The student will explain the pattern of decimal point movement when multiplying or dividing by powers of 10. The student will represent powers of 10 using exponents.	Students need to be provided with opportunities to explore this concept and come to this understanding; this should not just be taught procedurally. Example: Students might write: • 36 x 10 = 36 x 101 = 360 • 36 x 10 x 10 = 36 x 102 = 3600 • 36 x 10 x 10 x 10 = 36 x 103 = 36,000 • 36 x 10 x 10 x 10 x 10 = 36 x 104 = 360,000 Students might think and/or say: • I noticed that every time, I multiplied by 10 I added a zero to the end of the number. That makes sense because each digit's value became 10 times larger. To make a digit 10 times larger, I have to move it one place value to the left. • When I multiplied 36 by 10, the 30 became 300. The 6 became 60 or the 36 became 360. So I had to add a zero at the end to have the 3 represent 3 one-hundreds (instead of 3 tens) and the 6 represents 6 tens (instead of 6 ones). Students should be able to use the same type of reasoning as above to explain why the following multiplication and division problem by powers of 10 make sense. • 523 x 103 = 523,000 The place value of 523 is increased by 3 places. • 5.223 x 102 = 522.3 The place value of 5.223 is increased by 2 places. • 52.3 ÷ 101 = 5.23 The place value of 52.3 is decreased by one place.

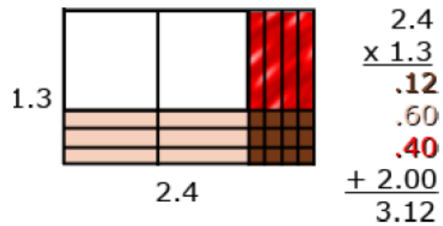
5.NBT.3. Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form [e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 (1/10) + 9 (1/100) + 2 (1/1000)]. b. Compare two decimals to thousandths place based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	a. The student will read and write decimals to the thousands using base-ten numerals, number names, and expanded form. b. The student will read, write, and compare decimals to thousandths place using symbols to record the results of the comparison	Students build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. They use concrete models and number lines to extend this understanding to decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc. They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation. This investigation leads them to understanding equivalence of decimals (0.8 = 0.80 = 0.800). Example: Some equivalent forms of 0.72 are: 72/100 7/10 + 2/100 7 x (1/10) + 2 x (1/100) 0.70 + 0.02 70/100 + 2/100 0.720 7 x (1/10) + 2 x (1/100) + 0 x (1/1000) 720/1000 Students need to understand the size of decimal numbers and relate them to common benchmarks such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals. Example: Comparing 0.25 and 0.17, a student might think, “25 hundredths is more than 17 hundredths”. They may also think that it is 8 hundredths more. They may write this comparison as 0.25 > 0.17 and recognize that 0.17 < 0.25 is another way to express this comparison. Comparing 0.207 to 0.26, a student might think, “Both numbers have 2 tenths, so I need to compare the hundredths. The second number has 6 hundredths and the first number has no hundredths so the second number must be larger. Another student might think while writing fractions, “I know that 0.207 is 207 thousandths (and may write 207/1000). 0.26 is 26 hundredths (and may write 26/100) but I can also think of it as 260 thousandths (260/1000). So, 260 thousandths is more than 207 thousandths.
5.NBT.4. Use place values understanding to round decimals to any place.	The student will use place value understanding to round a decimal to any place value.	Example: Round 14.235 to the nearest tenth. Students recognize that the possible answer must be in tenths thus, it is either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30). 
Perform operations with multi-digit whole numbers and with decimals to hundredths.		
5.NBT.5. Fluently multiply multi-digit whole numbers using a standard algorithm.	The student will fluently multiply multi-digit whole number using a standard algorithm.	In prior grades, students used various strategies to multiply. Students can continue to use these different strategies as long as they are efficient, but must also understand be able to use the standard algorithm. In applying the standard algorithm, students recognize the importance of place value.

		Example: 123 x 34. When students apply the standard algorithm, they, decompose 34 into 30 + 4. Then they multiply 123 by 4, the value of the number in the ones place, and then multiply 123 by 30, the value of the 3 in the tens place, and add the two products.
5.NBT.6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, number lines, real life situations, and/or area models.	The student will solve division problems with up to four-digit dividends and two-digit divisors with no remainders. The student will illustrate and explain the calculation using equations, rectangular arrays, number lines, real life situations, and/or area models.	
5.NBT.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between the operations. Relate the strategy to a written method and explain their reasoning in getting their answers.	The student will use concrete model or drawings and strategies to add, subtract, multiply, and divide decimals to hundredths. The student will explain and justify their model or drawing in words.	This standard requires students to extend the models and strategies they developed for whole numbers in grades 1-4 to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers. Examples: 3.6 + 1.7 A student might estimate the sum to be larger than 5 because 3.6 is more than 3 1/2 and 1.7 is more than 1 1/2 5.4 - 0.8 A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted. 6 x 2.4 A student might estimate an answer between 12 and 18 since 6 x 2 is 12 and 6 x 3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than 6 x 2 1/2 and think of 2 ½ groups of 6 as 12 (2 groups of 6) + 3 (½ of a group of 6). Students should be able to express that when they add decimals they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in fourth grade. Example: 4 - 0.3 • 3 tenths subtracted from 4 wholes. The wholes must be divided into tenths.  The answer is 3 and 7/10 or 3.7. Example 1.25 + 0.40 + 0.75 I saw that the 0.25 in 1.25 and the 0.75 would combine to equal 1

<table>
<tr>
<td></td>
<td></td>
<td>
whole. I then added the 2 wholes and the 0.40 to get 2.40

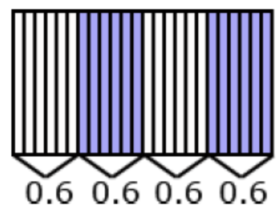


Example: An area model can be useful for illustrating products.



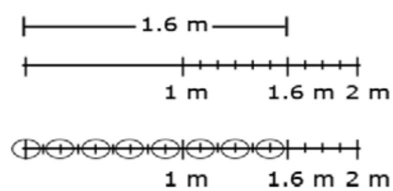
Students should be able to describe the partial products displayed by the area model. For example,
“3/10 times 4/10 is 12/100.
3/10 times 2 is 6/10 or 60/100.
1 group of 4/10 is 4/10 or 40/100.
1 group of 2 is 2.”

Example of division: finding the number in each group or share
Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as *2.4 ÷ 4 = 0.6*

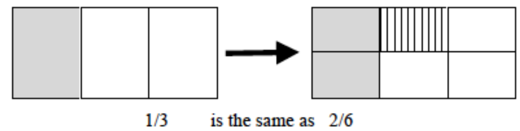
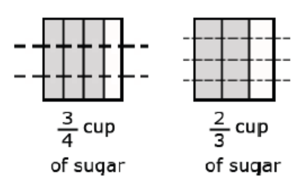
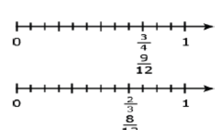
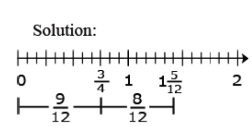


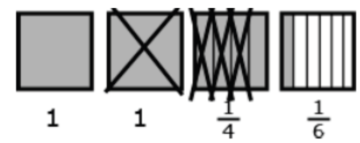
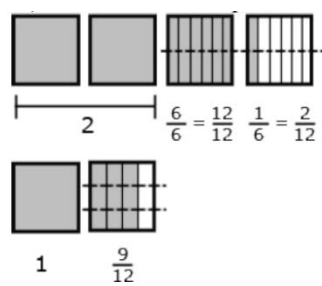
Example of division: find the number of groups
Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many can he cut?

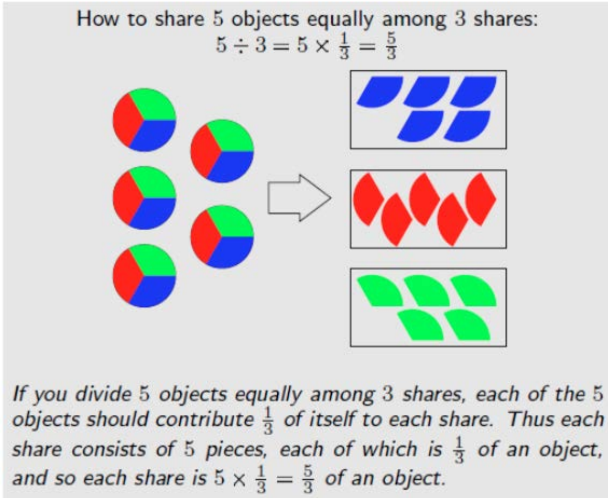
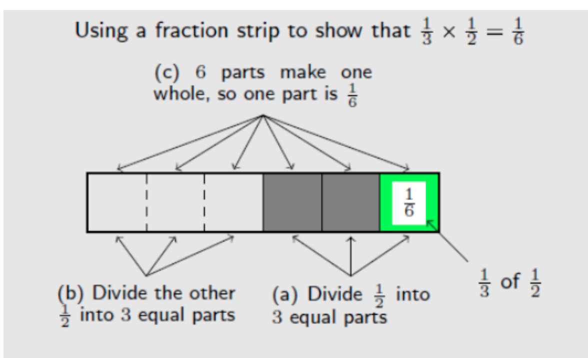
To divide to find the number of groups, a student might draw a segment to represent 1.6 meters. In doing so, s/he would count in tenths to identify the 6 tenths, and be able identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths.

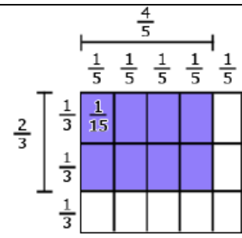


Students might count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as 10/10, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths, . . .16 tenths, a student can count 8 groups of 2 tenths.
Use their understanding of multiplication and think, “8 groups of 2 is 16, so 8 groups of 2/10 is 16/10 or 1 6/10.”
</td>
</tr>
<tr>
<td colspan="3">Numbers and Operations- Fractions</td>
</tr>
<tr>
<td colspan="3">Use equivalent fractions as a strategy to add and subtract fractions.</td>
</tr>
<tr>
<td>5.NF.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions</td>
<td>The student will model, draw, or build equivalent fractions, mixed numbers and improper fractions.</td>
<td>Students should apply their understanding of equivalent fractions developed in fourth grade and their ability to rewrite fractions in an equivalent form to find common denominators. They should know that multiplying the denominators will always give a common denominator but may not result in the smallest denominator.</td>
</tr>
</table>

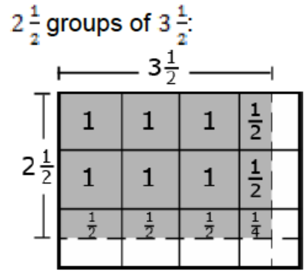
with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	The student will apply their understanding of equivalent fractions to rewrite fractions in an equivalent form to find common denominators. The student will show how their basic algorithm for finding common denominators works using a model. The student will use their understanding of equivalent fractions to add and subtract fractions of unlike denominators.	Example: 1/3 + 1/6  I drew a rectangle and shaded 1/3. I knew that if I cut every third in half then I would have sixths. Based on my picture, 1/3 equals 2/6. Then I shaded in another 1/6 with stripes. I ended up with an answer of 3/6, which is equal to ½. Fifth grade students will need to express both fractions in terms of a new denominator with adding unlike denominators. For example, in calculating 2/3 + 5/4 they reason that if each third in 2/3 is subdivided into fourths and each fourth in 5/4 is subdivided into thirds, then each fraction will be a sum of unit fractions with denominator 3 x 4 = 4 x 3 + 12: Example: $\frac{2}{3} + \frac{5}{4} = \frac{2 \times 4}{3 \times 4} + \frac{5 \times 3}{4 \times 3} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}.$ Example: $\frac{2}{5} + \frac{7}{8} = \frac{16}{40} + \frac{35}{40} = \frac{51}{40}$ $3\frac{1}{4} - \frac{1}{6} = 3\frac{3}{12} - \frac{2}{12} = 3\frac{1}{12}$
5.NF.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators (e.g., by using visual fraction models or equations to represent the problem). Use benchmark fractions and number sense of fractions to estimate mentally and check the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*	The student will use models and their understanding of unlike denominators and equivalent fractions to solve word problems. The student will use benchmark fractions and number sense to estimate mentally to check the reasonableness of their answers.	Example: Jerry was making two different types of cookies. One recipe needed 3/4 cup of sugar and the other needed 2/3 cup of sugar. How much sugar did he need to make both recipes? Mental estimation: A student may say that Jerry needs more than 1 cup of sugar but less than 2 cups. An explanation may compare both fractions to ½ and state that both are larger than ½ so the total must be more than 1. In addition, both fractions are slightly less than 1 so the sum cannot be more than 2.  $\frac{3}{4} = \frac{9}{12}$ $\frac{2}{3} = \frac{8}{12}$ $\frac{3}{4} + \frac{2}{3} = \frac{17}{12} = \frac{12}{12} + \frac{5}{12} = 1\frac{5}{12}$ • Area model Linear Model   Example Using a bar diagram: Sonia had 2 1/3 candy bars. She promised her brother that she would give him ½ of a candy bar. How much will she have left after she gives her brother the amount she promised?

		• If Mary ran 3 miles every week for 4 weeks, she would reach her goal for the month. The first day of the first week she ran 1 3/4 miles. How many miles does she still need to run the first week? Using addition to find the answer: 1 3/4 + n = 3 A student might add 1 ¼ to 1 ¾ to get to 3 miles. Then he or she would add 1/6 more. Thus 1 ¼ miles + 1/6 of a mile is what Mary needs to run during that week. Example: Using an area model to subtract • This model shows 1 ¾ subtracted from 3 1/6 leaving 1 + 1/4 = 1/6 which a student can then change to 1 + 3/12 + 2/12 = 1 5/12. 3 1/6 can be expressed with a denominator of 12. Once this is done a student can complete the problem, 2 14/12 – 1 9/12 = 1 5/12.  This diagram models a way to show how 3 1/6 and 1 ¾ can be expressed with a denominator of 12. Once this accomplished, a student can complete the problem, 2 14/12 – 1 9/12 = 1 5/12  Elli drank 3/5 quarts of milk and Javier drank 1/10 of a quart less than Ellie. How much milk did they drink all together? $\frac{3}{5} - \frac{1}{10} = \frac{6}{10} - \frac{1}{10} = \frac{5}{10}$ Solution: $\frac{3}{5} - \frac{1}{10} = \frac{6}{10} - \frac{1}{10} = \frac{5}{10}$ This is how much milk Javier drank. $\frac{3}{5} + \frac{5}{10} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10}$ Together they drank 1 1/10 quarts of milk. This solution is reasonable because Ellie drank more than 1/2 quart and Javier drank ½ quart so together they drank slightly more than one quart.
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.		
5.NF.3. Interpret a fraction as division of the numerator by the denominator (*a*/*b* = *a* ÷ *b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers (e.g., by using visual fraction models	The student will model the connection of fractions with division and explain this by showing division as equal sharing. The student will model and solve word problems involving division of whole	This standard calls for students to connect their understanding of partitioning a number line from third and fourth grade. Students need ample experiences to explore the concept that a fraction is a way to represent the division of two quantities. Students are expected to demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read 3/5 as “three fifths” and after many experiences with sharing problems,

or equations to represent the problem). *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*	numbers that lead to answers in the form of fractions or mixed numbers.	learn that 3/5 can also be interpreted as "3 divided by 5."  *(Progressions for the CCSSM, Number and Operation – Fractions, CCSS Writing Team, August 2011, page 11)* Example: Ten team members are sharing 3 boxes of cookies. How much of a box will each student get? When working this problem a student should recognize that the 3 boxes are being divided into 10 groups, so s/he is seeing the solution to the following equation, 10 x n = 3 (10 groups of some amount is 3 boxes) which can also be written as n = 3 ÷ 10. Using models or diagram, they divide each box into 10 groups, resulting in each team member getting 3/10 of a box.
5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show	The student will use a model to show the process of multiplying fractions or a whole number by a fraction. a. The student will justify the product of multiplying a whole number by a fraction using a drawing or number line. b. The student will justify the product of multiplying fractions using an area model drawing.	Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard.  *(Progression for the CCSSM, Number and Operation – Fractions, CCSS Writing Team, August 2011, page 11)*

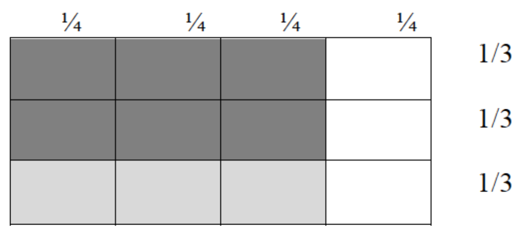
<table>
<tr>
<td>that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</td>
<td></td>
<td>


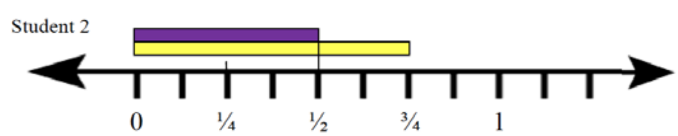
The area model and the line segments show that the area is the same quantity as the product of the side lengths.



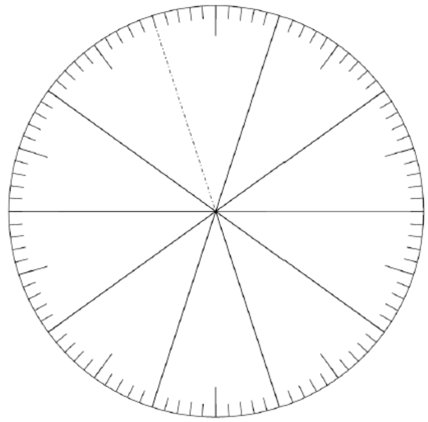
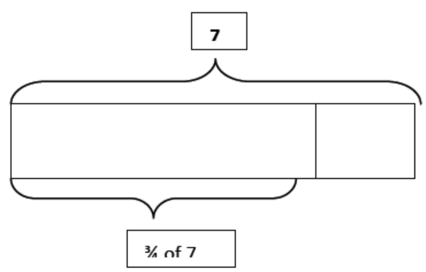
Example:
Three-fourths of the class is boys. Two-thirds of the boys are wearing tennis shoes. What fraction of the class are boys with tennis shoes?
This question is asking what 2/3 of ¾ is, or what is 2/3 x ¾. What is 2/3 x 3/4, in this case you have 2/3 groups of size ¾ (a way to think about it in terms of the language for whole numbers is 4 x 5 you have 4 groups of size 5.
The array model is very transferable from whole number work and then to binomials.

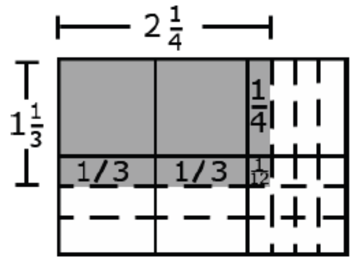
Student 1
I drew a rectangle to represent the whole class. The four columns represent the fourths of a class. I shaded 3 columns to represent the fraction that are boys. I then split the rectangle with horizontal lines into thirds. The dark area represents the fraction of the boys in the class wearing tennis shoes, which is 6 out of 12. That is 6/12, which equals 1/2.

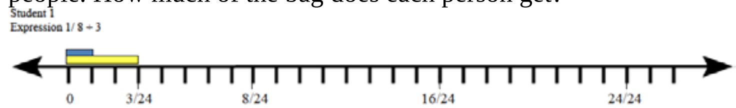
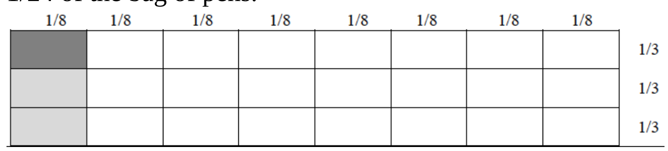
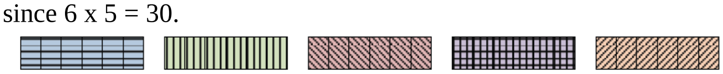
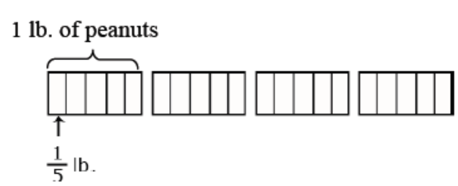


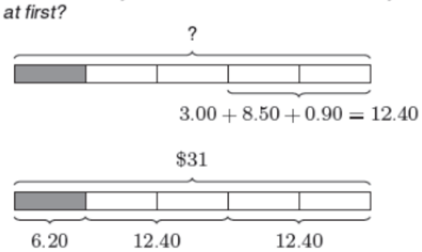


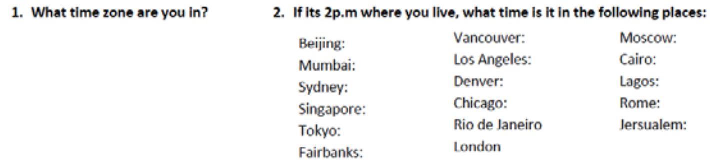
Student 3
Fraction circle could be used to model student thinking.
First I shade the fraction circle to show the ¾ and then overlay with 2/3 of that?
</td>
</tr>
</table>

		
5.NF.5. Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. (Division of a fraction by a fraction is not a requirement at this grade.)	a. The student will examine the products in terms of the relationship between two types of problems (in other words, if one factor changes what is the effect on the product in fractional terms) b. The student will justify the size of a product to the size of one factor on the basis of the size of the other factor relative to 1 (i.e. is the factor less than 1 or more than 1), without performing the indicated multiplication (in other words, students will predict why the product will be larger or smaller than one of the factors).	This standard calls for students to examine the magnitude of products in terms of the relationship between two types of problems. a. Example 1: Mrs. Jones teaches in a room that is 60 feet wide and 40 feet long. Mr. Thomas teaches in a room that is half as wide, but has the same length. How do the dimensions and area of Mr. Thomas' classroom compare to Mrs. Jones' room? Draw a picture to prove your answer. a. Example 2: How does the product of 225 x 60 compare to the product of 225 x 30? How do you know? Since 30 is half of 60, the product of 225x 60 will be double or twice as large as the product of 225 x 30. b. Example 1: ¾ of 7 is less than 7 because 7 is multiplied by a factor less than 1 so the product must be less than 7.  b. Example 2 2 2/3 x 8 must be more than 8 because 2 groups of 8 is16, and 2 2/3 is almost 3 groups of 8. So the answer must be close to, but less than 24. $\frac{3}{4} = \frac{5 \times 3}{5 \times 4}$ because multiplying $\frac{3}{4}$ by $\frac{5}{5}$ is the same as multiplying by 1.
5.NF.6. Solve real-world problems involving multiplication of fractions and mixed numbers (e.g., by using visual fraction models or equations to represent the problem).	The student will use multiple visual fraction models to solve real world problems involving multiplication of fraction and mixed numbers.	Example: Evan bought 6 roses for his mother 2/3 of them were red. How many red roses were there? Using a visual, a student divides the 6 roses into 3 groups and counts how many are in 2 of the 3 groups.  A student can use an equation to solve. $\frac{2}{3} \times 6 = \frac{12}{3} = 4$ red roses

		Example: Mary and Joe determined that the dimensions of their school flag needed to be ft. 1 1/3 by 2 1/4 ft. What will be the area of the school flag? A student can draw an array to find this product and can also use his or her understanding of decomposing numbers to explain the multiplication. Thinking ahead a student may decide to multiply by 1 1/3 instead of 2 1/4.  The explanation may include the following: First, I am going to multiply 2 ¼ by 1 and then by 1/3 When I multiply 2 ¼ by 1, it equals 2 ¼ Now I have to multiply 2 ¼ by 1/3 1/3 times 2 is 2/3 1/3 times ¼ is 1/12 So the answer is 2 ¼ + 2/3 + 1/12 or 2 3/12 + 8/12 + ½ = 2 12/12 = 3

5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.* b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.* c. Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions (e.g., by using visual fraction models and equations to represent the problem). *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*	The student will divide unit fractions by whole numbers. a. The student will use a visual model to compute the quotient of a fraction divided by a whole number (in a given story), and justify the answer using the inverse relationship between multiplication and division. b. The student will use a visual fraction model where the whole number is being divided by a unit fraction (in a given story). TSW then explain and defend their answer. c. The student will solve real world problems involving division of unit fractions by non-zero whole number and division of whole numbers by unit factions. Use visual fraction models and equations to represent the problem.	This is the first time that students are dividing fractions. In fourth grade students divided whole numbers, and multiplied a whole number by a fraction. The concept unit fraction is a fraction that has a one in the numerator. For example, the fraction 3/5 is 3 copies of the unit fraction 1/5. a. Example: You have 1/8 of a bag of pens and you need to share them among 3 people. How much of the bag does each person get?  Student 2 I drew a rectangle and divided it into 8 columns to represent my 1/8. I shaded the first column. I then needed to divide the shaded region into 3 parts to represent sharing among 3 people. I shaded one-third of the first column even darker. The dark shade is 1/24 of the grid or 1/24 of the bag of pens.  Student 3 1/8 of a bag of pens divided by 3 people. I know that my answer will be less than 1/8 since I'm sharing 1/8 into 3 groups. I multiplied 8 by 3 and got 24, so my answer is 1/24 of the bag of pens. I know that my answer is correct because (1/24) x 3 = 3/24 which equals 1/8. b. Example: The bowl holds 5 Liters of water. If we use a scoop that holds 1/6 of a Liter, how many scoops will we need in order to fill the entire bowl? Student: I created 5 boxes. Each box represents 1 Liter of water. I then divided each box into sixths to represent the size of the scoop. My answer is the number of small boxes, which is 30. That makes sense since 6 x 5 = 30.  1 = 1/6 + 1/6 + 1/6 + 1/6 + 1/6 a whole has 6/6 so five wholes would be 6/6 + 6/6 + 6/6 + 6/6 + 6/6 =30/6 c. Example Knowing how many in each group/share and finding how many groups/shares Angelo has 4 lbs of peanuts. He wants to give each of his friends 1/5 lb. How many friends can receive 1/5 lb of peanuts? A diagram for 4 ÷ 1/5 is shown below. Students explain that since there are five fifths in one whole, there must be 20 fifths in 4 lbs. 
Measurement and Data		
Convert like measurement units within a given measurement system and solve problems involving time.		

5.MD.1. Identify, estimate measure, and convert equivalent measures within systems English length (inches, feet, yards, miles) weight (ounces, pounds, tons) volume (fluid ounces, cups, pints, quarts, gallons) temperature (Fahrenheit) Metric length (millimeters, centimeters, meters, kilometers) volume (milliliters, liters), temperature (Celsius), (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real-world problems using appropriate tools.	The student will use appropriate tools strategically and precisely to find accurate measurements (both metric and customary). The student will convert among different-sized standard measurement units within a given measurement system (both metric and customary). The student will use conversions in solving multi-step, real world problems.	Both customary and standard measurement systems are included; students worked with both metric and customary units of length in second grade. In third grade, students work with metric units of mass and liquid volume. In fourth grade, students work with both systems and begin conversions within systems in length, mass and volume. Students should explore how the base-ten system supports conversions within the metric system. Example: 100 cm = 1 meter. In Grade 5, students extend their abilities from Grade 4 to express measurements in larger or smaller units within a measurement system. This is an excellent opportunity to reinforce notions of place value for whole numbers and decimals, and connection between fractions and decimals (e.g., 2 ½ meters can be expressed as 2.5 meters or 250 centimeters). For example, building on the table from Grade 4, Grade 5 students might complete a table of equivalent measurements in feet and inches. Grade 5 students also learn and use such conversions in solving multi-step, real world problems (see example below). Feet | Inches 0 | 0 | 1 | 2 | 3 | In Grade 6, this table can be discussed in terms of ratios and proportional relationships (see the Ratio and Proportion Progression). In Grade 5, however, the main focus is on arriving at the measurements that generate the table. **Multi-step problem with unit conversion** *Kumi spent a fifth of her money on lunch. She then spent half of what remained. She bought a card game for $3, a book for $8.50, and candy for 90 cents. How much money did she have at first?*  Students can use tape diagrams to represent problems that involve conversion of units, drawing diagrams of important features and relationships (MP1). *(Progressions for the CCSSM, Geometric Measurement, CCSS Writing Team, August 2011, page 26)*
5.MD.2. Solve real-world problems involving elapsed time between world time zones. (L)	The student will solve real-world problems involving elapsed time between world time zones.	Example: Timeline Practice Instructions- Using the map below answer the questions below

<table>
<tr>
<td></td>
<td></td>
<td>


1. What time zone are you in?
2. If its 2p.m where you live, what time is it in the following places:

Beijing:
Mumbai:
Sydney:
Singapore:
Tokyo:
Fairbanks:

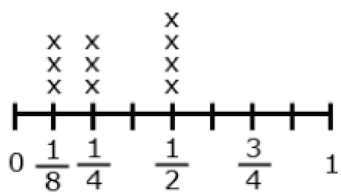
Vancouver:
Los Angeles:
Denver:
Chicago:
Rio de Janeiro
London

Moscow:
Cairo:
Lagos:
Rome:
Jersualem:

https://teacher.ocps.net/david.mcintosh/media/Time%20Zones%20Worksheet.pdf
</td>
</tr>
<tr>
<td colspan="3">Represent and interpret data.</td>
</tr>
<tr>
<td>5.MD.3. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</td>
<td>The student will make a line-plot that will display a set of data of measurements that include fractions. (1/2, ¼, 1/8).

The student will use the line plot to solve real-world problem.</td>
<td>Example:
Students measured the first ten words in a paragraph to the nearest 1/8 of an inch then displayed data collected on a line plot. How many words measured closest to 1/8, ¼, ½? If you put all the words together end to end what would be the total length of all the words?

Length of the First 10 Words



Length of Words in Inches</td>
</tr>
<tr>
<td>5.MD.4. Explain the classification of data from real-world problems shown in graphical representations including the use of terms mean and median with a given set of data. (L)</td>
<td>The student will determine the mean and median from a set of data.

The student will use the terms mean and median to interpret the set of data from a real-world problem.

The student will will explain which term, mean or median, represents the data accurately in a given situation.</td>
<td>Example:

Sean throws the dice 7 times. Here are his results:
4, 2, 1, 2, 4, 2, 6,

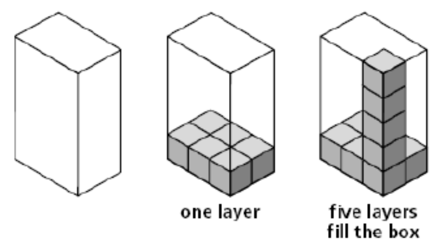
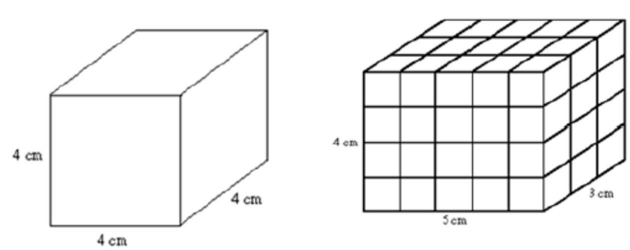
What is the mean? (4+2+1+2+4+2+6) ÷ 7 = 3

What is the median? Put the numbers in order of size and find the middle number. (Answer: 2)

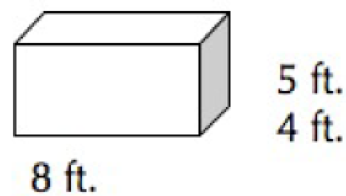
What is the mode? (Answer: 2)

http://www.gscdn.org/library/cms/62/13762.pdf

Example 2:
Find the mean, median, and mode of your last five math grades. Then explain which term, mean or median, represents your data more accurately?</td>
</tr>
<tr>
<td colspan="3">Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.</td>
</tr>
</table>

5.MD.5. Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure that can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.	The student will identify the attributes of a cubic unit The student will understand that cubic units are used to measure the volume of solid figures. The student will explain and demonstrate that volume is using cubic units to pack a solid figure without gaps or overlaps.	The concept of volume should be extended from area with the idea that students are covering an area (the bottom of cube) with a layer of unit cubes and then adding layers of unit cubes on top of bottom layer (see picture below). Students should have ample experiences with concrete manipulatives before moving to pictorial representations. Students' prior experiences with volume were restricted to liquid volume. As students develop their understanding volume they understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit. This cubic unit is written with an exponent of 3 (e.g., in3, m3).
5.MD.6. Estimate and measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units.	The student will estimate and measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and non-standard units.	Example: Students explore the number of unit cubes in a variety of containers and make estimations.
5.MD.7. Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. a. Estimate and find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Demonstrate the associative property of multiplication by using the product of three whole numbers to find volumes (length x width x height). b. Apply the formulas *V* = *l* × *w* × *h* and *V* = *b* × *h* for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. c. Recognize volume as additive. Find volumes of	The student will estimate and find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes. The student will label the measurements of the length, width, and height, of the figure The student will find the area of the base of the figure and show that the volume is equal to the area of the base multiplied by the height The student will demonstrate the associative property of multiplication by using the product of three whole numbers to find volumes (length x width x height). b. The student will apply the formulas V= l x w x h and V = b x h for rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. c. The student will find the volume of solid figures composed of two non-	Example: Student begins to relate volume to operation of multiplication and addition.  (3 x 2) represented by first layer (3 x 2) x 5 represented by number of 3 x 2 layers (3 x 2) + (3 x 2) + (3 x 2) + (3 x 2)+ (3 x 2) = 6 + 6 + 6 + 6 + 6 + 6 = 30 6 representing the size/area of one layer  b. Example: When given 24 cubes, students make as many rectangular prisms as possible with a volume of 24 cubic units. Students build the prisms and record possible dimensions.

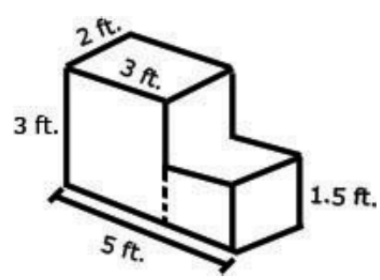
<table>
<tr>
<td>solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.</td>
<td>overlapping right rectangular prisms by adding the volumes of the non-overlapping parts and apply this technique to solve real-world problems.</td>
<td>
<table>
<tr><th>Length</th><th>Width</th><th>Height</th></tr>
<tr><td>1</td><td>2</td><td>12</td></tr>
<tr><td>2</td><td>2</td><td>6</td></tr>
<tr><td>4</td><td>2</td><td>3</td></tr>
<tr><td>8</td><td>3</td><td>1</td></tr>
</table>

b. Example:
Cari is the lead architect for the city’s new aquarium. All of the tanks in the aquarium will be rectangular prisms where the side lengths are whole numbers.
a. Cari’s first tank is 4 feet wide, 8 feet long and 5 feet high. How many cubic feet of water can her tank hold?


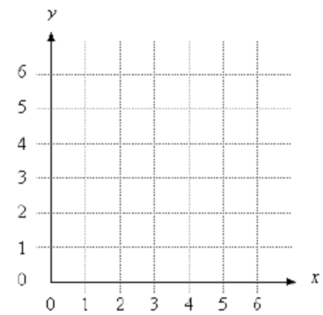
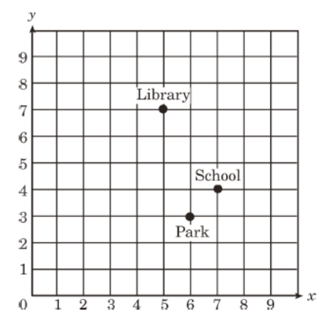
b. Cari knows that a certain species of fish needs at least 240 cubic feet of water in their tank. Create 3 separate tanks that hold exactly 240 cubic feet of water. (Ex: She could design a tank that is 10 feet wide, 4 feet long and 6 feet in height.)

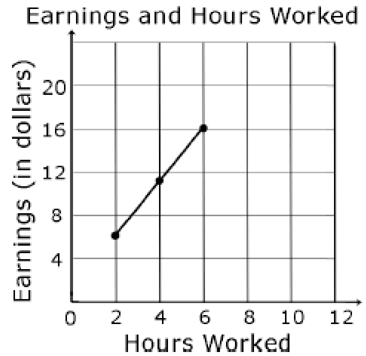
c. In the back of the aquarium, Cari realizes that the ceiling is only 10 feet high. She needs to create a tank that can hold exactly 100 cubic feet of water. Name one way that she could build a tank that is not taller than 10 feet.

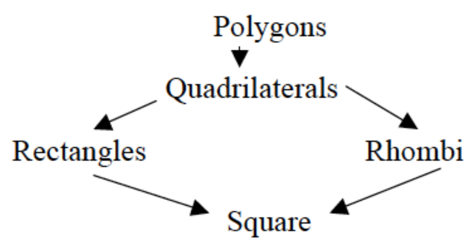
http://www.illustrativemathematics.org/illustrations/1308

c. Example:
Students determine the volume of concrete needed to build the steps in the diagram below.


</td>
</tr>
<tr><td colspan="3">Geometry</td></tr>
<tr><td colspan="3">Graph points on the coordinate plane to solve real-world and mathematical problems.</td></tr>
<tr>
<td>5.G.1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and</td>
<td>The student will create a coordinate plane using two perpendicular number lines called axes with the intersection of the number lines coincide with the 0 on each line.</td>
<td>The first number in the ordered pair represents how far to travel from the origin on the X (vertical) axis. The second number in the ordered pair represents how far to travel from the origin on the Y (horizontal) axis.</td>
</tr>
</table>

a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).	The student will demonstrate and explain that a given point in the plane is located by using an ordered pair of numbers, called its coordinates. The student will will demonstrate the plotting of a coordinate pair.	Example: Connect these points in order on the coordinate grid below: (2, 2) (2, 4) (2, 6) (2, 8) (4, 5) (6, 8) (6, 6) (6, 4) and (6, 2). **Coordinate Grid** What letter is formed on the grid? *Solution: "M" is formed.* 
5.G.2. Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	The student will Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	This standard references real-world and mathematical problems, including the traveling from one point to another and identifying the coordinates of missing points in geometric figures, such as squares, rectangles, and parallelograms. Example: Emanuel draws a line segment from (1, 3) to (8, 10). He then draws a line segment from (0, 2) to (7, 9). If he wants to draw another line segment that is parallel to those two segments what points will he use? Example: Using the coordinate grid, which ordered pair represents the location of the School? Explain a possible path from the school to the library.  Example: Sara has saved $20. She earns $8 for each hour she works. If Sara saves all of her money, how much will she have after working 3 hours? 5 hours? 10 hours? Create a graph that shows the relationship between the hours Sara worked and the amount of money she has saved. What other information do you know from analyzing the graph? Example: Use the graph below to determine how much money Jack makes after working exactly 9 hours.

		
Classify two-dimensional (plane) figures into categories based on their properties.		
5.G.3. Understand that attributes belonging to a category of two-dimensional (plane) figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*	The student will categorize two-dimensional (plane) figures based on their common attributes.	Geometric properties include properties of sides (parallel, perpendicular, congruent), properties of angles (type, measurement, congruent), and properties of symmetry (point and line). Example: If the opposite sides on a parallelogram are parallel and congruent, then rectangles are parallelograms. A sample of questions that might be posed to students include: ▪ A parallelogram has 4 sides with both sets of opposite sides parallel. What types of quadrilaterals are parallelograms? ▪ Regular polygons have all of their sides and angles congruent. Name or draw some regular polygons. ▪ All rectangles have 4 right angles. Squares have 4 right angles so they are also rectangles. True or False? ▪ A trapezoid has 2 sides parallel so it must be a parallelogram. True or False? Students can precisely describe, classify, and understand relationships among types of two dimensional objects using their defining properties at: http://illuminations.nctm.org/LessonDetail.aspx?ID=L277
5.G.4. Classify two-dimensional (plane) figures in a hierarchy based on attributes and properties.	The student will classify two-dimensional (plane) figures in a hierarchy based on attributes and properties.	Example: A kite is a quadrilateral whose four sides can be grouped into two pairs of equal-length sides that are beside (adjacent to) each other. Example: Create a Hierarchy Diagram using the following terms: polygons – a closed plane figure formed from line segments that meet only at their endpoints. quadrilaterals - a four-sided polygon. rectangles - a quadrilateral with two pairs of congruent parallel sides and four right angles. rhombi – a parallelogram with all four sides equal in length. square – a parallelogram with four congruent sides and four right angles.

		Possible student solution:  Here’s another way to look at how a fifth-grade teacher in Michigan has chosen to teach the hierarchy of quadrilaterals: http://msjohnson308.blogspot.com/2011/11/quadrilateral-hierarchy.html