# Geometry

| Unit 1: Congruence, Proof, and Constructions | | |
|---|---|---|
| In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent. In this unit, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal proof. Students prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work. | | |
| **Alaska Standard** | **Student Outcome for Mastery** | **Time** |
| **G.CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. | Understand and use definitions of angles, circles, perpendicular lines, parallel lines, and line segments based on the undefined term of a point, a line, the distance along a line, and the length of an arc. | |
| **G.CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). | Use various technologies to represent and compare rigid and size transformations of figures in a coordinate plane. Comparing transformations that preserve distance and angle to those that do not.<br>Describe and compare function transformations on a set of points as inputs to produce another set of points as outputs, to include translations and horizontal and vertical stretching. | |
| **G.CO.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. | Describe the rotations and reflections of a rectangle, parallelogram, trapezoid, or regular polygon that maps each figure onto itself. | |
| **G.CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. | Using previous comparisons and descriptions of transformations develop and understand the meaning of rotations, reflections, and translations based on angles, circles, perpendicular lines, parallel lines, and line segments. | |
| **G.CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, racing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. | Transform a geometric figure given a rotation, reflection, or translation using graph paper, tracing paper, or geometric software.<br>Create sequences of transformations that map a geometric figure on to itself and another geometric figure. | |
| **G.CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | Use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane.<br>Knowing that rigid transformations preserve size and shape or distance and angle, use this fact to connect the idea of congruency and develop the definition of congruent. | |
| **G.CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | Use the definition of congruence, based on rigid motion, to show two triangles are congruent if and only if their corresponding sides and corresponding angles are congruent. | |
| **G.CO.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. | Use the definition of congruence, based on rigid motion, to develop and explain the triangle congruence criteria; ASA, SSS, and SAS. Prove triangles are congruent. | |
| **G.CO.9** Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* | Prove and use theorems pertaining to lines and angles:<br>• Vertical angles<br>• Parallel lines cut by a transversal and resulting angles<br>• Perpendicular bisector of a line segment | |
| **G.CO.10** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* | Prove and use theorems pertaining to triangles:<br>• Sum of interior angles equal 180 degrees<br>• Isosceles triangles<br>• Mid-segments<br>• Centroid | |

| | | |
|---|---|---|
| **G.CO.11** Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* | Prove and use theorems pertaining to parallelograms:<br>• Angle and side congruence<br>• Diagonals of a parallelogram bisect each other<br>• Rectangles are parallelograms with congruent diagonals. | |
| **G.CO.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* | Make formal geometric constructions of:<br>• Copying a segment.<br>• Copying an angle.<br>• Bisecting a segment.<br>• Bisecting an angle.<br>• Perpendicular lines<br>• Line parallel to a given line through a point not on the line. | |
| **G.CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | Make formal geometric constructions of inscribed polygons including:<br>• equilateral triangle<br>• square<br>• regular hexagon | |
| **Unit 2: Similarity, Proof, and Trigonometry** | | |
| Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. | | |
| **Alaska Standard** | **Student Outcome for Mastery** | **Time** |
| **G.SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor.<br>a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.<br>b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. | When given a center and a scale factor, verify that when dilating a figure in a coordinate plane, a segment of the pre-image that does not pass through the center of the dilation, is parallel to its image when the dilation is performed. However, a segment that passes through the center remains unchanged.<br>When given a center and a scale factor, verify experimentally, that when performing dilations of a line segment, the pre-image, the segment which becomes the image is longer or shorter based on the ratio given by the scale factor. | |
| **G.SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | Use the idea of dilation transformations to develop the definition of similarity based on the equality of corresponding angles and the proportionality of corresponding sides. Given two figures, determine whether they are similar and explain their similarity. | |
| **G.SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | Use the properties of similarity transformations to develop the AA criteria for proving similar triangles. | |
| **G.SRT.4** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity*. | Prove and use theorems pertaining to similar triangles:<br>• AA, SAS, SSS<br>• Lines parallel to one side of a triangle divide the other two proportionally<br>• Pythagorean Theorem using triangle similarity. | |
| **G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | Use similarity theorems to prove that two triangles are congruent Prove geometric figures, other than triangles, are similar and/or congruent. | |
| **G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | Use a corresponding angle of similar right triangles; show that the relationships of the side ratios are the same, which leads to the definition of trigonometric ratios for acute angles. | |

| | | |
|---|---|---|
| **G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles. | Demonstrate the relationship between the sine of an acute angle and the cosine of its complement. | |
| **G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★ | Apply trigonometric ratios and Pythagorean Theorem to solve application problems involving right triangles. | |
| **G.MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* | Use the concept of density when referring to situations involving area and volume models, such as persons per square mile. | |
| **G.MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).* | Solve design problems by designing an object or structure that satisfies certain constraints, such as minimizing cost or working with a grid system based on ratios (i.e., The enlargement of a picture using a grid and ratios and proportions) | |
| **G.MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).* | For a triangle that is not a right triangle, draw an auxiliary line from a vertex, perpendicular to the opposite side and derive the formula, A=½ ab sin (C), for the area of a triangle, using the fact that the height of the triangle is, h=a sin(C). | |
| **G.SRT.9** (+) Derive the formula *A* = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | Prove and use the Laws of Sines and Cosines and the relationship among sides and angles of any triangle, such as sin(C)=(h/a). | |
| **G.SRT.10** (+) Prove the Laws of Sines and Cosines and use them to solve problems. | Apply the Laws of Sines and Cosines to find unknown measures in right triangles, non-right triangles, and real-world problems. | |
| **G.SRT.11** (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). | When given a center and a scale factor, verify that when dilating a figure in a coordinate plane, a segment of the pre-image that does not pass through the center of the dilation, is parallel to its image when the dilation is performed. However, a segment that passes through the center remains unchanged.<br>When given a center and a scale factor, verify experimentally, that when performing dilations of a line segment, the pre-image, the segment which becomes the image is longer or shorter based on the ratio given by the scale factor. | |

| **Unit 3: Extending to Three Dimensions** | | |
|---|---|---|
| Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line. | | |
| **Alaska Standard** | **Student Outcome for Mastery** | **Time** |
| **G.GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* | Understand how formulas for circumference of a circle, area of a circle and the volume of a cylinder, pyramid and cone relate to each figure.<br>• ***Area of a Circle*** - Understand the formula for the area of a circle can be shown using dissection arguments. First dissect portions of the circle like pieces of a pie. Arrange the pieces into a curvy parallelogram, then find the area of the parallelogram.<br>• ***Volume of a Cylinder*** - Build on understandings of circles and volume from previous grades/courses to find the volume of cylinders, finding the area of the base $\pi r^2$ and multiplying by the number of layers (the height) $V = \pi r^2/h$.<br>• ***Volume of a Cone*** - Show that the volume of a cylinder is 3 times the volume of a cone having the same base area and height or that the volume of a cone is 1/3 the volume of a cylinder having the same base area and height. $V=\pi r^2 h/3$<br>• ***Volume of a Pyramid*** - Show that the volume of a prism is 3 times the volume of a pyramid having the same base area and height or that the volume of a pyramid is 1/3 the volume of a prism having the same base area and height. $V = Bh/3$ where B=Area of the Base and H=height of the pyramid | |
| **G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ | Determine the volume of cylinders, pyramids, cones, and spheres with a variety of bases, including both right and oblique.<br>Computations should be made using both physical models with measurement tools and diagrams or descriptions with appropriate formulas.<br>Relate the slant height, height and radius of a cone using the Pythagorean Theorem. | |
| **G.GMD.4** Identify the shapes of two-dimensional cross-sections of three dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. | Intersect or slice three-dimensional objects such as prisms, cones, spheres, and cylinders using a plane to produce and identify two-dimensional objects such as rectangles, parabolas, ellipses, circles, and hyperbolas.<br>Rotate two-dimensional objects about a point (such as the origin) or a line will produce three-dimensional objects such as cylinders and parabolic discs, etc | |
| **G.MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* | Upon inspection of objects, describe the objects using the properties of geometric shapes and their measurements.<br>Example: hexagonal honeycombs, parabolic satellite dish, cylindrical brace for a shin, etc. | |

## Unit 4: Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines. Students continue their study of quadratics by connecting the geometric and algebraic definitions of the parabola.

| Alaska Standard | Student Outcome for Mastery | Time |
|---|---|---|
| **G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* | Use coordinate geometry to prove geometric theorems algebraically; such as prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle. | |
| **G.GPE.5** Prove the slope criteria for parallel and perpendicular lines and uses them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). | Using slope, prove lines are parallel or perpendicular<br>Find equations of lines based on certain slope criteria such as; finding the equation of a line parallel or perpendicular to a given line that passes through a given point. | |
| **G.GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. | Given two points, find the point on the line segment between the two points that divides the segment into a given ratio. | |
| **G.GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★ | Use coordinate geometry and the distance formula to find the perimeters of polygons and the areas of triangles and rectangles. | |
| **G.GPE.2** Derive the equation of a parabola given a focus and directrix. | Given a focus and directrix, derive the equation of a parabola.<br>Given a parabola, identify the vertex, focus, directrix, and axis of symmetry, noting that every point on the parabola is the same distance from the focus and the directrix. | |

## Unit 5: Circles With and Without Coordinates

In this unit, students prove basic theorems about circles, with particular attention to perpendicularity and inscribed angles, in order to see symmetry in circles and as an application of triangle congruence criteria. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane, and apply techniques for solving quadratic equations to determine intersections between lines and circles or parabolas and between two circles.

| Alaska Standard | Student Outcome for Mastery | Time |
|---|---|---|
| **G.C.1** Prove that all circles are similar. | Prove all circles are similar | |
| **G.C.2** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* | Describe the differences between central angles, inscribed angles, and tangent-tangent (circumscribed) angles and the formulas that relate the measure of the angle to the measure of the intercepted arcs.<br>Communicate and apply special relationships caused by inscribed angles (inscribed angles on a diameter, inscribed quadrilaterals)<br>Apply the relationship between a tangent and a radius drawn to the point of tangency. | |
| **G.C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | Prove, construct, and use the inscribed and circumscribed circles for triangles and quadrilaterals. | |
| **G.C.4** (+) Construct a tangent line from a point outside a given circle to the circle. | From a point outside of a circle, construct a line tangent to the circle using construction tools or computer software. | |
| **G.C.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. | Define radian measure as a ratio of an arc length to its radius.<br>Convert between degrees and radians.<br>Use similarity to calculate the length of an arc.<br>Derive the formula for the area of a sector. | |

| | | |
|---|---|---|
| **G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. | When given an equation in the form $ax^2 + bx + cy^2 + dy = e$, use completing the square to find the center and radius of the circle.<br>Derive the equation of a circle using the Pythagorean Theorem. | |
| **G.GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* | Use coordinates to prove circle concepts algebraically (i.e. that a point lies on a circle, that a line is tangent to a circle, find the equation of a circle given the center and a point on the circle, etc) | |
| **G.MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).* | Prove all circles are similar | |

## Unit 6: Applications of Probability

Building on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

| Alaska Standard | Student Outcome for Mastery | Time |
|---|---|---|
| **S.CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). | Define a sample space and events within the sample space.<br>Identify subsets from sample space given defined events, including unions, intersections and complements of events. | |
| **S.CP.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. | Identify two events as independent or not.<br>Explain properties of Independence and Conditional Probabilities in context. | |
| **S.CP.3** Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. | Define and calculate conditional probabilities.<br>Use the Multiplication Principal to decide if two events are independent and to calculate conditional probabilities. | |
| **S.CP.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* | Construct and interpret two-way frequency tables of data for two categorical variables.<br>Calculate probabilities from the table.<br>Use probabilities from the table to evaluate independence of two variables. | |
| **S.CP.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* | Recognize and explain the concepts of independence and conditional probability in everyday situations. | |
| **S.CP.6** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. | Calculate conditional probabilities using the definition: the conditional probability of A given B as the fraction of B's outcomes that also belong to A. Interpret the probability in context. | |

| | | |
|---|---|---|
| **S.CP.7** Apply the Addition Rule, *P(A* or *B)* = *P(A)* + *P(B)* – *P(A* and *B)*, and interpret the answer in terms of the model. | Explain the Addition Rule for probability using Venn Diagrams and use it to solve word problems.<br>Interpret the answer as it relates to the word problem. | |
| **S.CP.8** (+) Apply the general Multiplication Rule in a uniform probability model, *P(A* and *B)* = *P(A)P(B\|A)* = *P(B)P(A\|B)*, and interpret the answer in terms of the model. | Understand symbolic notation for probability.<br>Use the general Multiplication Rule to solve problems.(Example: using P(A and B) and P(A) to find *P(A\|B).)* and be able to explain the meaning of the answer as it relates to the problem. | |
| **S.CP.9** (+) Use permutations and combinations to compute probabilities of compound events and solve problems. | Solve compound probability problems using permutations and combinations. (Tree diagrams) | |
| **S.MD.6** (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). | Understand when a decision is fair (i.e. when the probabilities are equal) and use this to make truly random decisions.<br>Understand the applications of this to removing bias from experiments and surveys. | |
| **S.MD.7** (+) Analyze decisions and strategies using probability concepts (e.g., roduct testing, medical testing, pulling a hockey goalie at the end of a game). | Understand the meaning of probabilities as a decimal between zero and one where a number closer to one represents an event that is more likely and a number closer to zero represents an even that is less likely.<br>Use probabilities in given situations to judge the best decision. | |