Algebra 2

Unit 1: Polynomial, Rational and Radical Relationships

This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Key: K- Knowledge (Name, Identify, Describe)
R- Reasoning (Explain, Compare/Contrast, Predict) S- Skill (Use, Solve, Calculate)
P- Product (Create, Write, Present)

Alaska Standard	Student Outcome for Mastery	Time
N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	Know that every number is a complex number of the form $a \pm bi$, where a and b are real numbers. Know that the complex number $i^2 = -1$.	
N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	Apply the fact that the complex number $i^2 = -1$. Use the associative, commutative, and distribute properties, to add, subtract, and multiply complex numbers.	
N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.	Solve quadratic equations with real coefficients that have solutions of the form $a + bi$ and $a - bi$.	
N.CN.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.	Use polynomial identities to write equivalent expressions in the form of complex numbers	
NCN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	Understand number of complex solutions to a polynomial equation is the same as the degree of the polynomial. Show that this is true for a quadratic polynomial. (The Fundamental Theorem of Algebra)	
A.SSE.1 Interpret expressions that represent a quantity in terms of its context. **a.** Interpret parts of an expression, such as terms, factors, and coefficients. **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*	Identify the different parts of an expression and explain their meaning within the context of a problem. Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.	
A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms. Use factoring techniques such as common factors, grouping, the difference of two squares, the sum or difference of two cubes, or a combination of methods to factor completely. Simplify expressions including combining like terms, using the distributive property and other operations with polynomials.	

A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*	Develop the formula for the sum of a finite geometric series when the ratio is not 1 and use the formula to solve problems.	
A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	Understand and apply the fact that polynomials form a system closed under addition, subtraction, multiplication.	
A.APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only *if* $(x – a)$ is a factor of p(x).	Understand and apply the Remainder Theorem and relate it to solving by factoring. Understand that a is a root of a polynomial function if and only if x-a is a factor of the function.	
A.APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	Find the zeros of a polynomial when the polynomial is factored. Use the zeros of a function to sketch a graph of the function	
A.APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.	Understand that polynomial identities include but are not limited to the product of the sum and difference of two terms, the difference of two squares, the sum and difference of two cubes, the square of a binomial, etc. Prove polynomial identities and illustrate how they are used to determine numerical relationships such as: $25^2 = (20+5)^2 = 20^2 +2\bullet 20\bullet5+5^2$	
A.APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.	For small values of n, use Pascal's Triangle to determine the coefficients of the binomial expansion. Use the Binomial Theorem to find the nth term in the expansion of a binomial to a positive power	
A.APR.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	Write rational expressions in different forms by multiplying and dividing them using long division and synthetic division. Use technology for complicated examples to assist with building a broader conceptual understanding.	
A.APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	Simplify rational expressions by adding, subtracting, multiplying, or dividing. Understand that rational expressions are closed under addition, subtraction, multiplication, and division (by a nonzero expression).	
A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	Solve simple rational and radical equations in one variable and provide examples of how extraneous solutions arise	

A.REI.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	Explain why the intersection of $y = f(x)$ and $y = g(x)$ is the solution of $f(x) = g(x)$ for any combination of linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Find the solution(s) by: • Using technology to graph the equations and determine their point of intersection, • Using tables of values, or • Using successive approximations that become closer and closer to the actual value	
F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **c.** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	Graph functions expressed symbolically and show key features of the graph. Graph simple cases by hand, and use technology to show more complicated cases including: Polynomial functions, identifying zeros when factorable, and showing end behavior.	

Unit 2: Trigonometric Functions

Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

Alaska Standard	Student Outcome for Mastery	Time
F.TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	Understand radian measure of an angle as the length of the intercepted arc on the unit circle. Compute the measure of an angle in degrees and radians.	
F.TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	Explain concepts related to the unit circle such as the relationship between: -real numbers and points on the circle -central angles, arc length, and radian measure -points on the unit circle and related trig. functions	
F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	Use sine and cosine to model periodic events such as the ocean's tide or the rotation of a Ferris wheel. Given the amplitude, frequency, and midline determine a trigonometric function used to model the situation.	
F.TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.	Using definitions for sin, cos, and tan, show that $\sin^2(\theta) + \cos^2(\theta) = 1$. Given values for sin(θ), cos(θ) or tan(θ) find other missing values. Determine the quadrant of an angle using the sign of the trig. function.	

Unit 3: Modeling with Functions		
In this unit students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. *The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.*		
Alaska Standard	**Student Outcome for Mastery**	**Time**
A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	Create and solve (quadratic, rational, and exponential) equations and inequalities in one variable.	
A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	Create and graph (quadratic, rational, and exponential) equations and inequalities in two variables.	
A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	Model, solve, and graph systems of equations or inequalities to represent given real-world contexts; Determine a feasible region and test intersection points using a constraint function;	
A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations; Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency - mentally or with paper and pencil in simple cases and using technology in all cases; Determine the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.	
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*	Identify, graph, and interpret key features of a function including: -x and y-intercepts -relative max and mins -increasing or decreasing intervals -end behavior -symmetry -periodicity	
F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*	State and justify the appropriate domain of a function.	
F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	Calculate the average rate of change from a variety of situations. i.e. from the graph, or from 2 points. Interpret the average rate of change over a specific interval. Use a graph to estimate the rate of the change.	

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **e.** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	Graph functions beyond the standard linear and quadratic functions, including square roots, cube roots, piecewise, step functions, absolute value functions, exponential, logarithmic and trigonometric functions.	
F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	Write a function in equivalent forms and explain the different properties revealed.	
F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	Compare the key features of two functions represented in different ways. For example, compare the end behavior of two functions, one represented graphically and the other represented symbolically.	
F.BF.1 Write a function that describes a relationship between two quantities. **b**. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	Write an expression, define a recursive process, or describe the calculations needed to model a function between two quantities. Combine function types (such as linear and exponential) using arithmetic operations. Compose functions.	
F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	Apply and describe transformations of functions. Given $f(x)$, describe in words and/or graphically $f(x) + k$, $kf(x)$, $f(kx)$, $f(x+k)$ Given $f(x)$ and a transformation of $f(x)$, determine the value of k. Recognize even and odd functions from both graphs and equations.	
F.BF.4 Find inverse functions. **a.** Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.	Solve a function for the dependent variable and write the inverse of a function by interchanging the values of the dependent and independent variables. Verify that one function is the inverse of another by illustrating that $f^{-1}(f(x)) = f(f^{-1}(x)) = x$. Read values of an inverse function from a graph or table. Find the inverse of a function that is not one-to-one by restricting the domain.	
F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	Express logarithms as solutions to exponential functions using bases 2, 10, and e. Use technology to evaluate a logarithm.	

Unit 4: Inferences and Conclusions from Data		
In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.		
Alaska Standard	**Student Outcome for Mastery**	**Time**
S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	Use the mean and standard deviation of a data set to fit it to a normal distribution, use the information to make estimations in regards to a population. Determine if a data set can/or should be fit to a normal distribution. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	
S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	Understand when it is appropriate to make inferences about a population from a sample.	
S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	Determine if a theoretical model is consistent with an experimental model based on data generated from the experiment.	
S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	Identify studies as a survey, experiment, or observational study. Discuss the appropriateness of each one's use in contexts with limiting factors. Design or evaluate sample surveys, experiments and observational studies with randomization. Discuss the importance of randomization in these processes.	
S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	Estimate the mean and proportions of a large population by testing a smaller population. Calculate a margin of error using simulations models.	
S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	Determine if differences in results from a randomized experiment are statistically significant.	
S.IC.6 Evaluate reports based on data.	Evaluate a report based on its data, source, and representation.	
S.MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	
S.MD/7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	Understand the meaning of given and calculated probabilities and use them to make and analyze decisions.	