AP Calculus AB - Summer Assignment - Solutions

1.) a) $$\frac{5(x+h)^2-5x^2}{h} = \frac{5(x+h)(x+h)-5x^2}{h} = \frac{5(x^2+2xh+h^2)-5x^2}{h}$$

$$= \frac{\cancel{5x^2}+10xh+5h^2-\cancel{5x^2}}{h} = \frac{10xh+5h^2}{h} = \frac{\cancel{h}(10x+5h)}{\cancel{h}} = \boxed{10x+5h}$$

$$\boxed{10x+5h}$$

b.) $$\frac{(x-1)^2(3x-1)-2(x-1)\cdot 3}{(x-1)^4} = \frac{(x-1)(3x-1)-6}{(x-1)^3}$$

Cancel out a common $(x-1)^1$ in both numerator & denominator.

$$= \frac{3x^2-x-3x+1-6}{(x-1)^3} = \boxed{\frac{3x^2-4x-5}{(x-1)^3}}$$

c.) $$\frac{\frac{a}{b}-a}{a+\left(\frac{a}{b}\right)^2} = \frac{\left(\frac{a}{b}-a\right)}{\left(a+\frac{a^2}{b^2}\right)} \cdot \frac{\frac{b^2}{1}}{\frac{b^2}{1}} = \frac{ab-ab^2}{ab^2+a^2}$$

$$= \frac{\cancel{a}(b-b^2)}{\cancel{a}(b^2+a)} = \boxed{\frac{b-b^2}{b^2+a}}$$

d.) $$\frac{2x(x+1)^2-3(x+1)^3}{8x^3+14x^2+6x} = \frac{2x(x+1)^2-3(x+1)^3}{2x(4x^2+7x+3)} = \frac{2x(x+1)^2-3(x+1)^3}{2x(4x+3)(x+1)}$$

$$= \frac{2x(x+1)-3(x+1)^2}{2x(4x+3)} = \frac{2x^2+2x-3(x^2+2x+1)}{2x(4x+3)}$$

$$= \frac{2x^2+2x-3x^2-6x-3}{2x(4x+3)} = \boxed{\frac{-x^2-4x-3}{2x(4x+3)}}$$

e) $\dfrac{y(-1)-(-x)\left(-\frac{x}{y}\right)}{y^2} = \dfrac{\left(-y-\frac{x^2}{y}\right)}{\left(y^2\right)} \cdot \dfrac{\frac{y}{1}}{\frac{y}{1}} = \boxed{\dfrac{-y^2-x^2}{y^3}}$

f.) $\sqrt[3]{x^3+8} = \sqrt[3]{(x+2)(x^2-2x+4)}$ $\Rightarrow$ No simplification possible.

$= \boxed{\sqrt[3]{x^3+8}} \neq x+2!$

2.) a) $\dfrac{3x+5}{(x-1)(x^4+7)} = 0$ when Numerator $=0 \rightarrow 3x+5=0$

$\boxed{x=-5/3}$

b) $(2x+1)(x-1)^2+(x+5)(2x+1)^2=0$

Factor out common $(2x+1)^1$: $(2x+1)\left[(x-1)^2+(x+5)(2x+1)\right]=0$

$= (2x+1)\left[(x^2-2x+1)+2x^2+11x+5\right]=0$

$= (2x+1)(3x^2+9x+6)=0$

$= 3(2x+1)(x^2+3x+2)=0$

$= 3(2x+1)(x+2)(x+1)=0$

$\boxed{x=-1/2, -2, -1}$

3.) $xy+xyz^2-3=5y+xz$

$xy+xyz^2-5y=xz+3$

$y(x+xz^2-5)=xz+3$

$\boxed{y=\dfrac{xz+3}{x+xz^2-5}}$

4.) a) $\frac{u+1}{u} = \frac{u}{u} + \frac{1}{u} = \boxed{1 + \frac{1}{u}}$

b.) $\frac{u\sqrt{u} + \sqrt[3]{u} + 1}{\sqrt{u}} = \frac{u^1 \cdot u^{1/2} + u^{1/3} + 1}{u^{1/2}} = \frac{u^1 \cdot u^{1/2}}{u^{1/2}} + \frac{u^{1/3}}{u^{1/2}} + \frac{1}{u^{1/2}}$

$= u + u^{1/3 - 1/2} + u^{-1/2} = \boxed{u + u^{-1/6} + u^{-1/2}}$

c) $\frac{5x^3 - 2\sqrt{x} - 4}{x^2} = \frac{5x^3}{x^2} - \frac{2x^{1/2}}{x^2} - \frac{4}{x^2}$

$= \boxed{5x - 2x^{-3/2} - 4x^{-2}}$

5.) a) $f(-4) = \boxed{-2}$ $\quad g(3) = \boxed{4}$

b) $f(x) = g(x)$ when graphs intersect, so @ $\boxed{x = -2, 2.}$

c) $\boxed{x = -3}$

d) f is dec. on $(0,4)$; f is inc. on $(-4,0)$
(negative slope) (positive slope)

e) D: $[-4, 4]$
R: $[-2, 3]$

f.) D: $[-4, 3]$
R: $[1/2, 4]$
approximate, so could be $(1/2, 4]$

6.) $\boxed{f(x) = x - x^2}$

a) $f(2+h) = 2+h-(2+h)^2 = 2+h-(4+4h+h^2)$

$= 2+h-4-4h-h^2 = \boxed{-h^2-3h-2}$

b.) $f(x+h) = x+h-(x+h)^2 = x+h-(x^2+2xh+h^2)$

$= \boxed{x+h-x^2-2xh-h^2}$

c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{x+h-(x+h)^2-(x-x^2)}{h}$

$= \dfrac{\cancel{x}+h-(x^2+2xh+h^2)-\cancel{x}+x^2}{h} = \dfrac{h-\cancel{x^2}-2xh-h^2+\cancel{x^2}}{h}$

$= \dfrac{h-2xh-h^2}{h} = \dfrac{\cancel{h}(1-2x-h)}{\cancel{h}} = \boxed{1-2x-h}$

7.) $f(x) = \dfrac{x}{x+1}$

a) $f(2+h) = \dfrac{2+h}{2+h+1} = \boxed{\dfrac{2+h}{3+h}}$

b) $f(x+h) = \boxed{\dfrac{x+h}{x+h+1}}$

c) $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\left(\dfrac{x+h}{(x+h+1)} - \dfrac{x}{(x+1)}\right)}{h} \cdot \dfrac{\dfrac{(x+h+1)(x+1)}{1}}{(x+h+1)(x+1)}$

$= \dfrac{(x+h)(x+1)-x(x+h+1)}{h(x+h+1)(x+1)} = \dfrac{\cancel{x^2}+\cancel{x}+\cancel{hx}+h-\cancel{x^2}-\cancel{xh}-\cancel{1x}}{h(x+h+1)(x+1)}$

$= \dfrac{h}{h(x+h+1)(x+1)} = \boxed{\dfrac{1}{(x+h+1)(x+1)}}$

8) $f(x) = \frac{2}{3}x^{3/2}$

$$f(16) - f(4) = \frac{2}{3}(16)^{3/2} - \frac{2}{3}(4)^{3/2} = \frac{2}{3}(\sqrt{16})^3 - \frac{2}{3}(\sqrt{4})^3$$

$$= \frac{2}{3}(4)^3 - \frac{2}{3}(2)^3 = \frac{2}{3}\cdot 64 - \frac{2}{3}\cdot 8 = \frac{128}{3} - \frac{16}{3}$$

$$= \boxed{\frac{112}{3}}$$

9.) a) $f(x) = \frac{x^4}{x^2+x-6} = \frac{x^4}{(x+3)(x-2)}$ ← Denominator $\neq 0$,

so $x \neq -3, 2$ or $\boxed{(-\infty,-3),(-3,2),(2,\infty)}$

Domain

b.) $f(x) = \sqrt{7-3x}$ ← $7-3x \geq 0$ or else is imaginary.

$-3x \geq -7$

$\boxed{x \leq \frac{7}{3}}$ or $\boxed{(-\infty, 7/3]}$

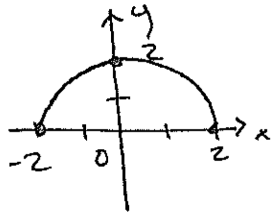
c) $f(x) = \sqrt[3]{x-1}$ ← $(-\infty, \infty)$ since cubic roots may be positive, negative, or 0.

d) $\ln x + 3$ ← If $\ln(x+3)$, then $x+3>0$ or $\boxed{x>-3}$

↖ If $\ln(x)+3$, then $\boxed{x>0}$

10.) a) $f(x) = \sqrt{4-x^2}$

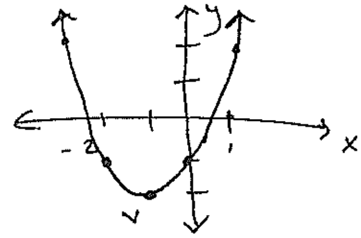
D: $[-2, 2]$



b.) $f(x) = x^2+2x-1$

Axis: $x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1 \rightarrow \boxed{x=-1}$

$V(-1, -2)$ $f(-1) = (-1)^2 + 2(-1) - 1 = 1-2-1 =$

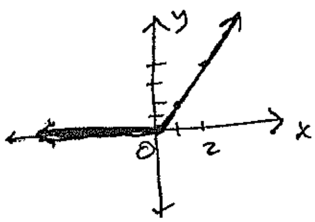


$\boxed{D: (-\infty, \infty)}$

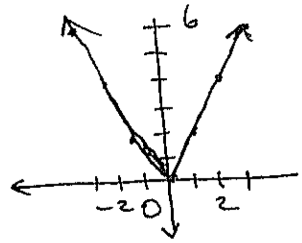
D : $\mathbb{R}$

c) $f(x) = |x| + x$

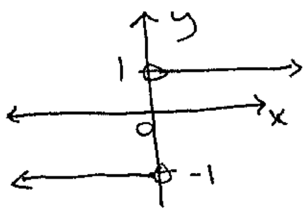
D: $(-\infty, \infty)$



d) $f(x) = |2x|$ D: $(-\infty, \infty)$



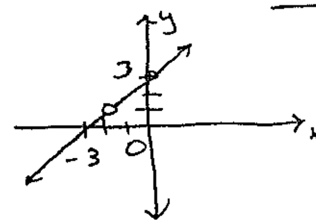
e.) $f(x) = \dfrac{x}{|x|}$ D: $(-\infty, 0), (0, \infty)$



f.) $f(x) = \dfrac{x^2+5x+6}{x+2}$

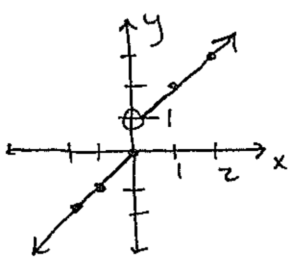
$f(x) = \dfrac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}} = x+3$

So, there is a hole at $x=-2$.



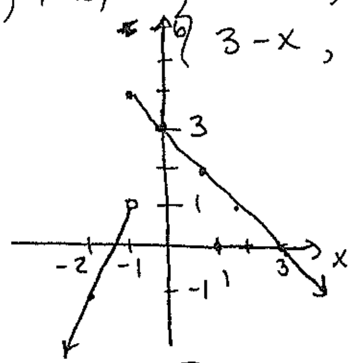
g.) $f(x) = \begin{cases} x, & x \le 0 \\ x+1, & x > 0 \end{cases}$

D: $(-\infty, 0), (0, \infty)$

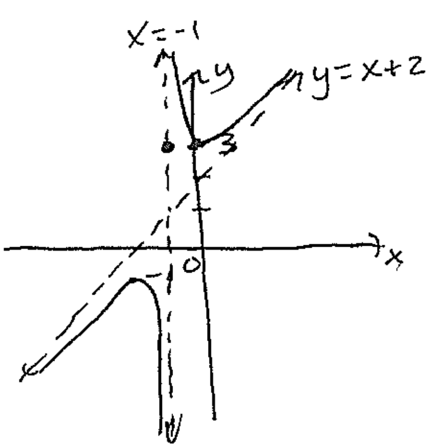


h.) $f(x) = \begin{cases} 2x+3, & x < -1 \\ 3-x, & x \ge -1 \end{cases}$

D: $(-\infty, -1), (-1, \infty)$

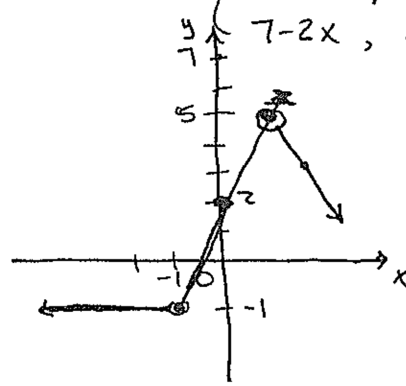


i.) $f(x) = \begin{cases} \dfrac{x^2+3x+3}{x+1}, & x \ne -1 \\ 3, & x = -1 \end{cases}$



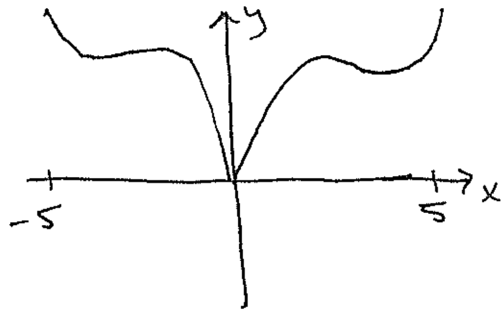
D: $(-\infty, \infty)$

j.) $f(x) = \begin{cases} -1, & x \le -1 \\ 3x+2, & -1 < x < 1 \\ 7-2x, & x \ge 1 \end{cases}$

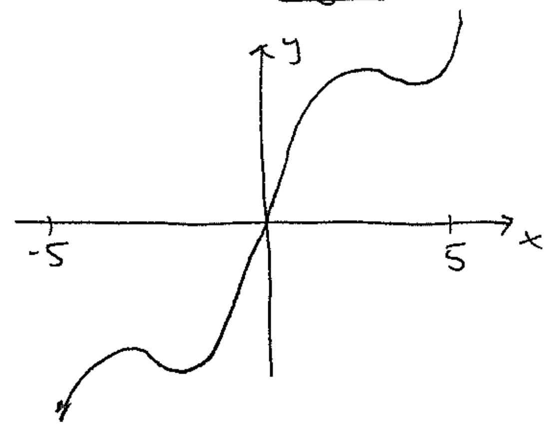


D: $(-\infty, \infty)$

11.) a) even functions have y-axis symmetry.



b.) odd functions have symmetry across the origin.



12.) a) $y = 5f(x)$ vertical stretch of 5.

b) $y = f(x-5)$ right 5 (horizontal shift of 5)

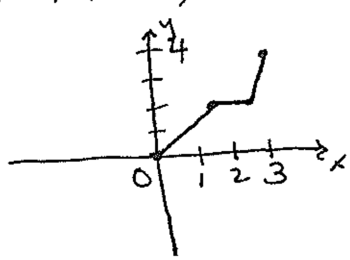
c) $y = -f(x)$ reflected across x-axis.

d) $y = f(5x)$ horizontal compression of $\frac{1}{5}$.

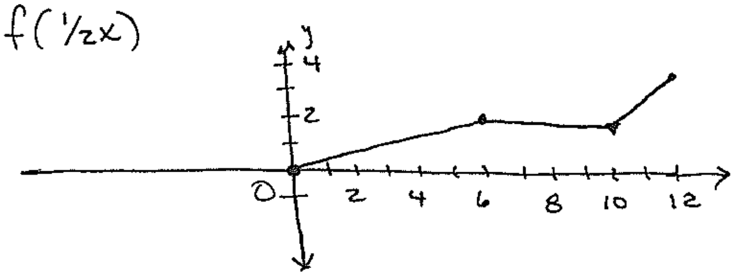
e) $y = f(x+5) - 3$ left 5, down 3 (horiz. shift of 5, vertical shift of -3)

f.) $y = f(-x)$, reflected across y-axis.

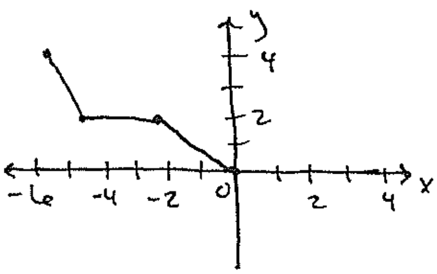
13.) a) $f(2x)$



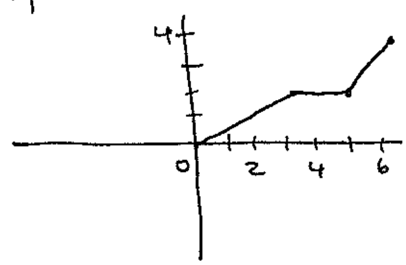
b.) $f(\frac{1}{2}x)$



c.) $f(-x)$



d.) $|f(x)|$



14.) $f(x)=\sqrt{x^2-16}$, $f(x)=3$?

$3=\sqrt{x^2-16}$

$3^2=\left(\sqrt{x^2-16}\right)^2$

$9=x^2-16$

$25=x^2$

$x=\pm\sqrt{25}=\boxed{\pm 5}$

15.) $f(x)=\dfrac{4-x^2}{x^2+x}=0$ when $4-x^2=0$

or $x^2=4$

$x=\pm\sqrt{4}$

$\boxed{x=\pm 2}$

16.) $f(x)=\sin\left(\frac{x}{2}\right)-3$, find $f(\pi/3)$.

$f(\pi/3)=\sin\left(\frac{\pi/3}{2}\right)-3=\sin(\pi/6)-3=1/2-3=\boxed{-2\tfrac{1}{2}}$

17.) $f(x)=\cos^2 3x-\sqrt{2}$, find $f(2\pi/9)$

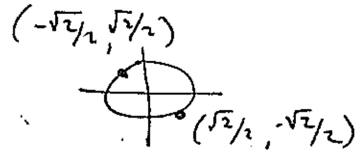
$f(2\pi/9)=\left(\cos\left(3\cdot 2\pi/9\right)\right)^2-\sqrt{2}=\left(\cos(2\pi/3)\right)^2-\sqrt{2}$

$=\left(-\frac{1}{2}\right)^2-\sqrt{2}=\boxed{\frac{1}{4}-\sqrt{2}}$

18.) $f(x)=\tan x+1=0$

$\tan x=-1$

$x=\tan^{-1}(-1/1)=-45^\circ$ or $-\pi/4$



$\Rightarrow \boxed{x=3\pi/4,\ 7\pi/4}$

19.) $f(x)=\sin^{-1}e^x$, $f(0)=\sin^{-1}(e^0)=\sin^{-1}(1)=\boxed{\pi/2}$

20.) $f(x)=\ln x-\cos^{-1}(x)$, find $f(1)$.

$f(1)=\ln 1-\cos^{-1}(1)$

$f(1)=0-0=\boxed{0}$

21.) $g(x)=-3f(x+1)-2x+3$, $g(2)=$?

$g(2)=-3f(2+1)-2(2)+3$

$g(2)=-3f(3)-4+3$

$\boxed{g(2)=-3f(3)-1}$

22.) $m = -2/3$, $(4,-1)$ $\rightarrow$ $y - y_1 = m(x - x_1)$

$\boxed{y + 1 = -2/3(x-4)}$ or $y + 1 = -2/3x + 8/3$

$\boxed{y = -2/3x + 5/3}$

23.) $2x + 3y = 4$, $(-1, 6)$

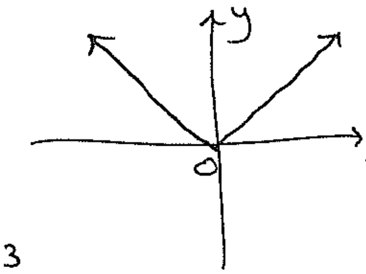
<u>parallel</u> $m_1 = m_2$. (Same slopes)

$3y = -2x + 4 \rightarrow y = -2/3x + 4/3$, So $\boxed{m = -2/3}$

Now: $\boxed{y - 6 = -2/3(x+1)}$ or $y = -2/3x - 2/3 + 6$

$\boxed{y = -2/3x + 16/3}$

24.) a) $f(x) = |x| \longrightarrow f(x) = \begin{cases} -x, & x < 0 \\ 0, & x = 0 \\ x, & x > 0 \end{cases}$



or $= \begin{cases} -x, & x < \\ x, & x \geq \end{cases}$

24.) b) $f(x) = |x+3| \rightarrow f(x) = \begin{cases} -x-3, & x < -3 \\ 0, & x = -3 \\ x+3, & x > -3 \end{cases}$ or $= \begin{cases} -x-3, & x < -3 \\ x+3, & x \geq -3 \end{cases}$

24.) c) $f(x) = |6 + 8x| \rightarrow f(x) = \begin{cases} -6-8x, & x < -3/4 \\ 6+8x, & x \geq -3/4 \end{cases}$

25.) $f(x) = x^2 + 8x - 5$, $g(x) = 3x - 1$.

$g(2) = 3(2) - 1 = \boxed{5}$

$f(2) = (2)^2 + 8(2) - 5 = 4 + 16 - 5 = \boxed{15}$

$f(g(2)) - g(f(2)) = f(5) - g(15)$

$= 25 + 40 - 5 - (45 - 1) = 60 - 44 = \boxed{16}$

26.) $f(x) = x^2 - 4x + 3$, $g(x) = 2x - 1$.

$f(g(x)) = (2x-1)^2 - 4(2x-1) + 3 = 4x^2 - 4x + 1 - 8x + 4 + 3 = \boxed{4x^2 - 12x + 8}$

$g(f(x)) = 2(x^2 - 4x + 3) - 1 = 2x^2 - 8x + 6 - 1 = \boxed{2x^2 - 8x + 5}$

27.) a) $f(x) = 4x-2$

$$y = 4x - 2 \rightarrow x = 4y - 2$$

$$\frac{x+2}{4} = \frac{4y}{4}$$

$$y = \frac{1}{4}x + \frac{1}{2} \rightarrow \boxed{f^{-1}(x) = \frac{1}{4}x + \frac{1}{2}}$$

b.) $f(x) = e^{2x-2} \rightarrow y = e^{2x-2} \rightarrow f^{-1}: \; x = e^{2y-2}$

$$\ln x = \cancel{\ln e}^{2y-2}$$

$$\ln x = 2y - 2$$

$$\ln x + 2 = 2y$$

$$y = \boxed{f^{-1}(x) = \frac{\ln x + 2}{2}}$$

c) $f(x) = \frac{x+2}{x-3} \rightarrow y = \frac{x+2}{x-3} \rightarrow f^{-1}: \; x = \frac{y+2}{y-3}$

$$x(y-3) = y+2$$

$$xy - 3x = y + 2$$

$$xy - y = 3x + 2$$

$$y(x-1) = 3x + 2$$

$$y = \frac{3x+2}{x-1}$$

$$\boxed{f^{-1}(x) = \frac{3x+2}{x-1}}$$

28.) $y = x^2 + 10x + 20$

Axis: $x = \frac{-b}{2a} = \frac{-10}{2(1)} = -5 \rightarrow \boxed{x = -5}$

x	y
-5	$(-5)^2 + 10(-5) + 20 = -5$

$$\boxed{V(-5,-5)}$$

29.) a) $f(x) = \frac{2-x}{x^2-9}$

horizontal: $y = \lim_{x \to \infty} \frac{2-x}{x^2-9} \sim \frac{-x}{x^2} \sim \frac{-1}{x} \rightarrow \frac{-1}{\infty} \rightarrow 0 \quad \boxed{y=0}$

vertical: $x^2 - 9 = 0 \rightarrow (x+3)(x-3) = 0$

$$\boxed{x = -3, \; x = 3}$$

29.) b) $f(x)=\dfrac{x-2}{x+3}$ horiz: $y=\lim\limits_{x\to\infty}\dfrac{x-2}{x+3}\sim\dfrac{x}{x}\sim 1\to \boxed{y=1}$

vert: $x+3=0 \to \boxed{x=-3}$

c) $f(x)=\dfrac{x+3}{x^2-9}=\dfrac{\cancel{(x+3)}\,1}{\cancel{(x+3)}(x-3)}=\dfrac{1}{x-3}$

horiz: $\lim\limits_{x\to\infty}\dfrac{x+3}{x^2-9}\sim\dfrac{1}{x}\to\dfrac{1}{\infty}\to 0 \quad \boxed{y=0}$

vert: $x-3=0 \quad \boxed{x=3}$ NOT $x=-3$, since removable discontinuity.

30.) $f(x)=\dfrac{x^2-4x-6}{x+1}$

$$\begin{array}{r|l} & x-5-\frac{1}{x+1} \\ (x+1) & x^2-4x-6 \\ & -\ x^2+x \\ \hline & -5x-6 \\ & +5x+5 \\ \hline & -1 \end{array}$$

$-\dfrac{1}{x+1}\to\dfrac{-1}{\infty}\to 0$ as $x\to\infty$.

Slant: ~~y = lim x~~ $\boxed{y=x-5}$

31.) $(0,10)$, $(5,5)$

$(0,10)$: $y=Ce^{kx}\to 10=Ce^0\to\boxed{C=10}$

$(5,5)$: $y=10e^{kx}\to 5=10e^{5k}\to\dfrac{5}{10}=e^{5k}\to\dfrac{1}{2}=e^{5k}$

$\ln(.5)=\cancel{\ln e}^{5k}$

$\ln(.5)=5k$

$\boxed{k=\dfrac{\ln(.5)}{5}}$

32.) a) $\dfrac{\cos x}{\cot x}=\cos x\tan x=\cancel{\cos x}\cdot\dfrac{\sin x}{\cancel{\cos x}}=\boxed{\sin x}$

b) $\csc x-\cot x\cos x=\dfrac{1}{\sin x}-\dfrac{\cos x}{\sin x}\cdot\cos x=\dfrac{1-\cos^2 x}{\sin x}=\dfrac{\sin^2 x}{\sin x}=\boxed{\sin x}$

c) $\dfrac{\sin 2x}{\sin x}=\dfrac{2\cancel{\sin x}\cos x}{\cancel{\sin x}}=\boxed{2\cos x}$ So, $\boxed{C}$

33) a) $\sin^2 x + \cos^2 x = \boxed{1}$

b.) $\sin^3 x \cot^4 x \sec x$

$\sin x \cdot \sin^2 x \cdot \cot^2 x \cdot \cot^2 x \cdot \frac{1}{\cos x}$

$\cancel{\sin x} \cdot \cancel{\sin^2 x} \cdot \frac{\cos^2 x}{\cancel{\sin^2 x}} \cdot \frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cancel{\cos x}}$

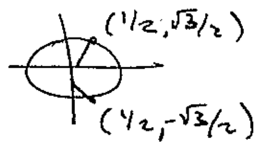
$\boxed{B}$ $= \boxed{\frac{\cos^3 x}{\sin x}} = \frac{\cos^2 x \cdot \cos x}{\sin x} \ldots$

c) $(\tan x - \sec x)(\tan x + \sec x) = \tan^2 x - \sec^2 x$

$\boxed{C}$ $= \sec^2 x - 1 - \sec^2 x = \boxed{-1}$

34.) $2\cos x - 1 = 0$

$\cos x = 1/2$



$x = \cos^{-1}(1/2)$

$\boxed{x = \pi/3,\ 5\pi/3}$

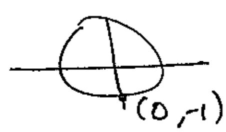
35.) $2\sin^2 x + \sin x = 1$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = 1/2$ | $\sin x = -1 \rightarrow x = \sin^{-1}(-1) = 3\pi/2$

$x = \sin^{-1}(1/2)$



$\boxed{x = \pi/6,\ 5\pi/6}$ $\boxed{x = \frac{3\pi}{2}}$

36.) a) $g(f(3)) = g(5) = \boxed{4}$

b) $g^{-1}(f^{-1}(1)) = g^{-1}(5) = \boxed{2}$

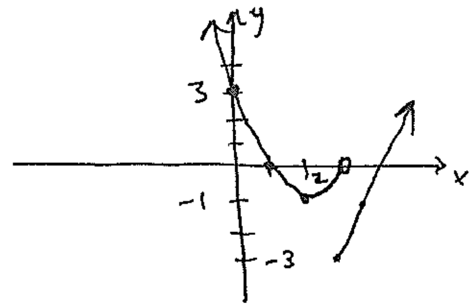
37.) $h(x) = \begin{cases} x^2 - 4x + 3, & x < 3 \\ 2x - 9, & x \geq 3 \end{cases}$

a) $h(0) = 0^2 - 4(0) + 3 = \boxed{3}$

b) $h(3) = 2(3) - 9 = \boxed{-3}$

c) $h(5) = 2(5) - 9 = \boxed{1}$

d.) min. value of $h(x) = \begin{cases} x^2-4x+3, & x<3 \\ 2x-9, & x \geq 3 \end{cases}$



$h(3) = \boxed{-3}$ min

$\longrightarrow h(x) = 8 = x^2-4x+3, \; x<3$

$0 = x^2-4x-5$

$(x-5)(x+1) = 0$

$x = \cancel{5}, -1 \quad \boxed{x=-1}$

or

$\longrightarrow h(x) = 8 = 2x-9, \; x \geq 3$

$17 = 2x$

$\boxed{x = 17/2}$

38.) $y = \#$ of gal. $\quad x = \#$ of hours

$(g(t) = y) \quad (t = \text{time})$

$(0, 360), (3, 354)$

$m = \dfrac{354-360}{3-0} = \dfrac{-6}{3} = \boxed{-2}$

$y = -2x + 360$

or $\boxed{g(t) = -2t + 360}$

b) $g(20) = -2(20) + 360 = -40 + 360$

$= \boxed{320 \text{ gal}}$

c) Need 360 - 320 gal = 40 gal to fill back up at a rate of 4 gal/hr. $\quad 40 \text{gal} \times \dfrac{1 \text{hr}}{4 \text{gal}} = \boxed{10 \text{ hrs}}$

or $h(t) = 4t + 320$

$360 = 4t + 320$

$40 = 4t$

$\boxed{t = 10}$

39.) $g(x) = \begin{cases} \frac{\text{[illegible]}}{2}, & x \text{ [illegible] } 4 \\ \sqrt{x}, & 0 < x < 4 \\ x^2, & x < 0 \end{cases}$

a) $g(-3) = (-3)^2 = \boxed{9}$ b) $g(1) = \sqrt{1} = \boxed{1}$ c) $g(0)$ = undefined
(not defined)

d.) $\boxed{\text{no}}$

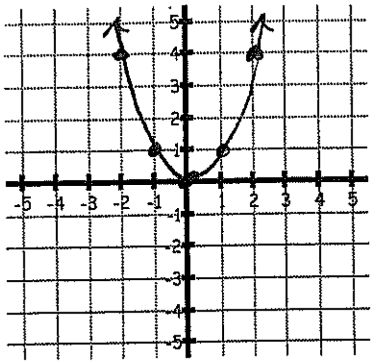
40.) a) $\log_2 16 = \log_2 2^4 = \boxed{4}$ b.) $\log_3 1 = \boxed{0}$ c) $\log 10 = \boxed{1}$

d) $\ln 1 = \boxed{0}$ e) $\ln e = \boxed{1}$ f.) $\ln e^3 = \boxed{3}$

Part 2 – Things that need to be memorized!

Graphs of Functions

$f(x) = x^2$



Max: —

Min: (0,0)

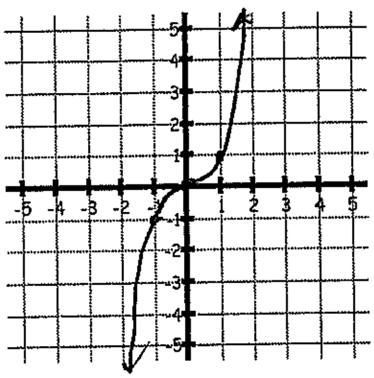
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = \infty$

$f(x) = x^3$



Max: —

Min: —

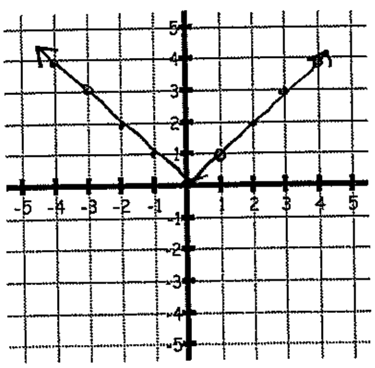
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

$f(x) = |x|$



Max: —

Min: (0,0)

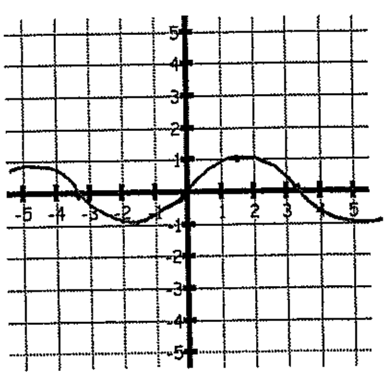
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = \infty$

$f(x) = \sin x$



Max: $(\pi/2, 1)$

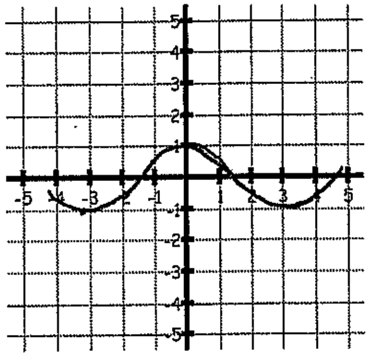
Min: $(3\pi/2, -1)$

Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

Period: 2π

$f(x) = \cos x$

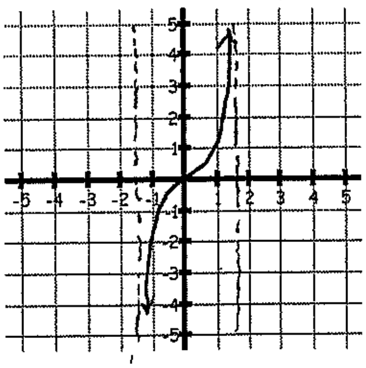


Max: $(0, 1)$ Min: $(\pi, -1)$

Domain: $(-\infty, \infty)$ Range: $[-1, 1]$

Period: 2π

$f(x) = \tan x$



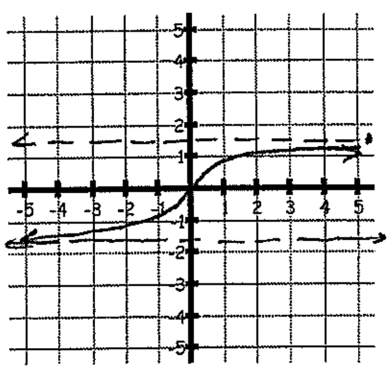
Max: — Min: —

Domain: $x \neq -\frac{\pi}{2} + n\pi$ Range: $(-\infty, \infty)$

Period: π

$\lim_{x \to \pi/2^-} f(x) = -\infty$ $\lim_{x \to \pi/2^+} f(x) = \infty$

$f(x) = \tan^{-1} x$

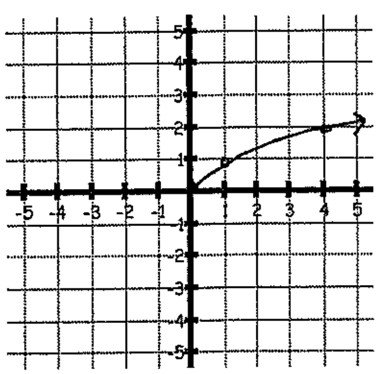


Domain: $(-\infty, \infty)$ Range: ~~$(-\pi/2, \pi/2)$~~ $y \neq -\frac{\pi}{2} + \pi n$

Hor. Asymptote(s): $y = -\frac{\pi}{2} + \pi n$

$\lim_{x \to \infty} f(x) = \pi/2$ $\lim_{x \to -\infty} f(x) = -\pi/2$

$f(x) = \sqrt{x}$

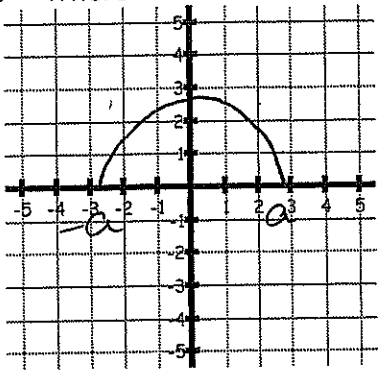


Max: — Min: $(0, 0)$

Domain: $[0, \infty)$ Range: $[0, \infty)$

$\lim_{x \to \infty} f(x) = \infty$ $\lim_{x \to 0^+} f(x) = 0$

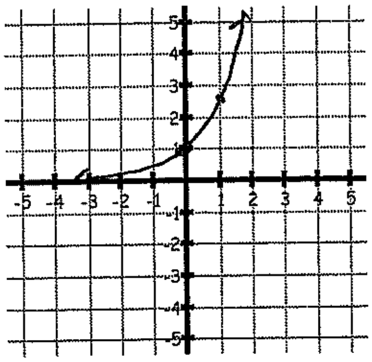
$f(x)=\sqrt{a^2-x^2}$ where "a" is a constant



Max: $(0,a)$ Min: $(-a,0),(a,0)$

Domain: $[-a,a]$ Range: $[0,a]$

$f(x)=e^x$



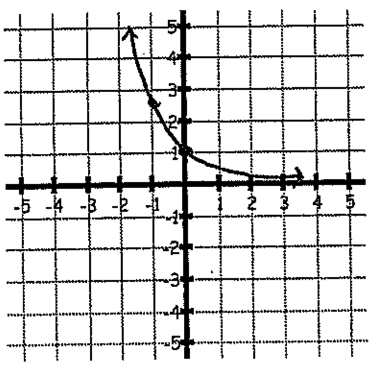
Max: — Min: —

Domain: $(-\infty,\infty)$ Range: $(0,\infty)$

Hor. Asymptote: $y=0$

$\lim\limits_{x\to\infty} f(x)= \infty$ $\lim\limits_{x\to-\infty} f(x)= 0$

$f(x)=e^{-x}$



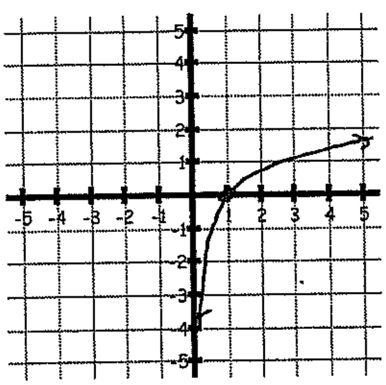
Max: — Min: —

Domain: $(-\infty,\infty)$ Range: $(0,\infty)$

Hor. Asymptote: $y=0$

$\lim\limits_{x\to\infty} f(x)= 0$ $\lim\limits_{x\to-\infty} f(x)= \infty$

$f(x)=\ln x$



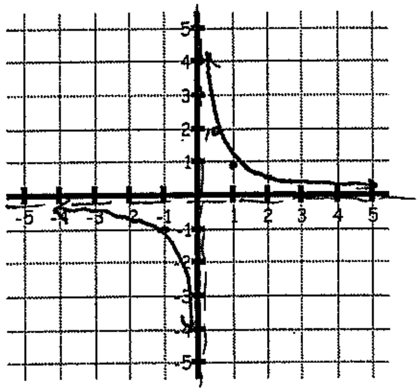
Max: — Min: —

Domain: $(0,\infty)$ Range: $(-\infty,\infty)$

Vert. Asymptote: $x=0$

$\lim\limits_{x\to\infty} f(x)= \infty$ $\lim\limits_{x\to0^+} f(x)= -\infty$

$f(x) = \frac{1}{x}$



Max: —

Min: —

Domain: $x \neq 0$

Range: $y \neq 0$

Hor. Asym: $y = 0$

Vert. Asym: $x = 0$

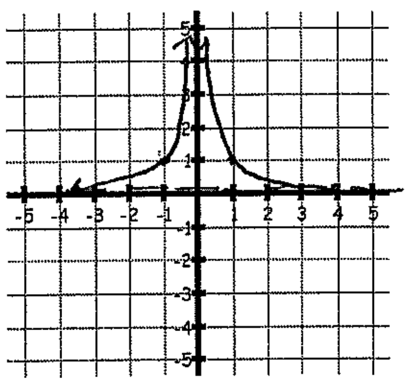
$\lim_{x\to-\infty} f(x) =$ 0

$\lim_{x\to\infty} f(x) =$ 0

$\lim_{x\to 0^+} f(x) =$ ∞

$\lim_{x\to 0^-} f(x) =$ $-\infty$

$f(x) = \frac{1}{x^2}$



Max: —

Min: —

Domain: $x \neq 0$

Range: $(0, \infty)$

Hor. Asym: $y = 0$

Vert. Asym: $x = 0$

$\lim_{x\to-\infty} f(x) =$ 0

$\lim_{x\to\infty} f(x) =$ 0

$\lim_{x\to 0^+} f(x) =$ ∞

$\lim_{x\to 0^-} f(x) =$ ∞

Exponential and Logarithmic Properties

$\ln 1 =$ 0 $\quad \ln e =$ 1 $\quad \ln e^2 =$ 2 $\quad \ln e^a =$ a

$\ln ab =$ $\ln a + \ln b$ $\quad \ln\left(\frac{a}{b}\right) =$ $\ln a - \ln b$

$\ln a^b =$ $b \ln a$

Trig Values

$\sin 0 =$ 0 $\quad \cos 0 =$ 1 $\quad \tan 0 =$ 0

$\sin\left(\frac{\pi}{6}\right) =$ $1/2$ $\quad \cos\left(\frac{\pi}{6}\right) =$ $\sqrt{3}/2$ $\quad \tan\left(\frac{\pi}{6}\right) =$ $\sqrt{3}/3$

$\sin\left(\frac{\pi}{4}\right) =$ $\sqrt{2}/2$ $\quad \cos\left(\frac{\pi}{4}\right) =$ $\sqrt{2}/2$ $\quad \tan\left(\frac{\pi}{4}\right) =$ 1

$\sin\left(\frac{\pi}{3}\right) = \underline{\frac{\sqrt{3}}{2}}$ $\cos\left(\frac{\pi}{3}\right) = \underline{\frac{1}{2}}$ $\tan\left(\frac{\pi}{3}\right) = \underline{\sqrt{3}}$

$\sin\left(\frac{\pi}{2}\right) = \underline{1}$ $\cos\left(\frac{\pi}{2}\right) = \underline{0}$ $\tan\left(\frac{\pi}{2}\right) = \underline{\text{und.}}$

$\sin\left(\frac{5\pi}{6}\right) = \underline{1/2}$ $\cos\left(\frac{5\pi}{6}\right) = \underline{-\sqrt{3}/2}$ $\tan\left(\frac{5\pi}{6}\right) = \underline{-\sqrt{3}/3}$

$\sin\left(\frac{3\pi}{4}\right) = \underline{\sqrt{2}/2}$ $\cos\left(\frac{3\pi}{4}\right) = \underline{-\sqrt{2}/2}$ $\tan\left(\frac{3\pi}{4}\right) = \underline{-1}$

$\sin\left(\frac{2\pi}{3}\right) = \underline{\sqrt{3}/2}$ $\cos\left(\frac{2\pi}{3}\right) = \underline{-1/2}$ $\tan\left(\frac{2\pi}{3}\right) = \underline{-\sqrt{3}}$

$\sin \pi = \underline{0}$ $\cos \pi = \underline{-1}$ $\tan \pi = \underline{0}$

$\sin\left(\frac{7\pi}{6}\right) = \underline{-1/2}$ $\cos\left(\frac{7\pi}{6}\right) = \underline{-\sqrt{3}/2}$ $\tan\left(\frac{7\pi}{6}\right) = \underline{\sqrt{3}/3}$

$\sin\left(\frac{5\pi}{4}\right) = \underline{-\sqrt{2}/2}$ $\cos\left(\frac{5\pi}{4}\right) = \underline{-\sqrt{2}/2}$ $\tan\left(\frac{5\pi}{4}\right) = \underline{1}$

$\sin\left(\frac{4\pi}{3}\right) = \underline{-\sqrt{3}/2}$ $\cos\left(\frac{4\pi}{3}\right) = \underline{-1/2}$ $\tan\left(\frac{4\pi}{3}\right) = \underline{\sqrt{3}}$

$\sin\left(\frac{3\pi}{2}\right) = \underline{-1}$ $\cos\left(\frac{3\pi}{2}\right) = \underline{0}$ $\tan\left(\frac{3\pi}{2}\right) = \underline{\text{und.}}$

$\sin\left(\frac{11\pi}{6}\right) = \underline{-1/2}$ $\cos\left(\frac{11\pi}{6}\right) = \underline{\sqrt{3}/2}$ $\tan\left(\frac{11\pi}{6}\right) = \underline{-\sqrt{3}/3}$

$\sin\left(\frac{7\pi}{4}\right) = \underline{-\sqrt{2}/2}$ $\cos\left(\frac{7\pi}{4}\right) = \underline{\sqrt{2}/2}$ $\tan\left(\frac{7\pi}{4}\right) = \underline{-1}$

$\sin\left(\frac{5\pi}{3}\right) = \underline{-\sqrt{3}/2}$ $\cos\left(\frac{5\pi}{3}\right) = \underline{1/2}$ $\tan\left(\frac{5\pi}{3}\right) = \underline{-\sqrt{3}}$

$\sin x$ is positive in Quadrants I, II

$\cos x$ is positive in Quadrants I, IV

$\tan x$ is positive in Quadrants I, III

Trig Identities

Pythagorean Identities:

$\sin^2 x + \cos^2 x =$ 1 $\qquad 1 + \cot^2 x =$ $\csc^2 x$ $\qquad \tan^2 x + 1 =$ $\sec^2 x$

Double Angle Identities:

$\sin 2x =$ $2\sin x \cos x$

$\cos 2x =$ $\cos^2 x - \sin^2 x$ $=$ $2\cos^2 x - 1$ $=$ $1 - 2\sin^2 x$

Reciprocal Identities:

$\csc x =$ $\frac{1}{\sin x}$ $\qquad \sec x =$ $\frac{1}{\cos x}$ $\qquad \cot x =$ $\frac{1}{\tan x}$

Inverse Trig Values

$\sin^{-1}\left(\frac{1}{2}\right) =$ $\pi/6$ $\qquad \cos^{-1}\left(\frac{1}{2}\right) =$ $\pi/3$ $\qquad \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) =$ $\pi/6$

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) =$ $\pi/4$ $\qquad \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) =$ $\pi/4$ $\qquad \tan^{-1} 1 =$ $\pi/4$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ $\pi/3$ $\qquad \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ $\pi/6$ $\qquad \tan^{-1} \sqrt{3} =$ $\pi/3$

$\sin^{-1} 0 =$ 0 $\qquad \cos^{-1} 0 =$ $\pi/2$ $\qquad \tan^{-1} 0 =$ 0

$\sin^{-1} 1 =$ $\pi/2$ $\qquad \cos^{-1} 1 =$ 0 $\qquad \tan^{-1} \infty =$ $\pi/2$

$\sin^{-1}\left(-\frac{1}{2}\right) =$ $-\pi/6$ $\qquad \cos^{-1}\left(-\frac{1}{2}\right) =$ $2\pi/3$ $\qquad \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) =$ $-\pi/6$

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ $-\pi/4$ $\qquad \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ $3\pi/4$ $\qquad \tan^{-1} -1 =$ $-\pi/4$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ $-\pi/3$ $\qquad \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ $5\pi/6$ $\qquad \tan^{-1} -\sqrt{3} =$ $-\pi/3$

$\sin^{-1} -1 =$ $-\pi/2$ $\qquad \cos^{-1} -1 =$ π $\qquad \tan^{-1} -\infty =$ $-\pi/2$

Part 3: An Introduction to Limits

The limit of a function is the y-value that you are getting close to as x gets close to some number in the domain. We write $\lim_{x \to a} f(x)$, which is read "the limit of $f(x)$ as x approaches a". The limit must be the same as x approaches "a" on both the left and the right.

There are many ways to find a limit. We will focus on three main ways for now: from a graph, from a table, and by direct substitution. Study the following examples of each type and then try the sample exercises on your own.

To find the limit from a table, look at the y-values as the x values get closer and closer to your "a" value. See if there is one number that all y-values seem to be going towards.

Example: Find $\lim_{x \to 2} x^2$.

X	1.9	1.99	1.999	2.001	2.01	2.1
Y	3.61	3.9601	3.996	4.004	4.0401	4.41

$\longrightarrow 4 \longleftarrow$

Solution: We are looking for the y-value as our x-values get closer and closer to 2. Looking at the chart from both the left and right sides, we can see that our y-values are getting closer and closer to 4. Therefore, $\lim_{x \to 2} x^2 = 4$.

Your turn: Find the following limits using the charts.

1. $\lim_{x \to 3} x^2 - 1$.

X	2.9	2.99	2.999	3.001	3.01	3.1
Y	7.41	7.9401	7.994	8.006	8.0601	8.61

$\boxed{8}$

2. $\lim_{x \to -1} 2x$.

X	-0.9	-0.99	-0.999	-1.001	-1.01	-1.1
Y	-1.8	-1.98	-1.998	-2.002	-2.02	-2.2

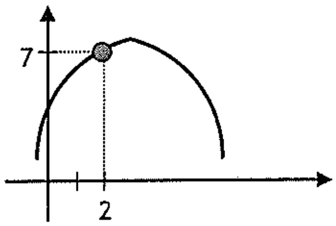
$\boxed{2}$

Solution: 1. 8; 2. -2

To find a limit from a graph, we follow the graph on either side of our "a" value towards that "a" value. The answer will be whatever y-value our graph is approaching. It is important to note that the graph does not have to actually "hit" that y-value. A limit is simply "what the y-value should be." If there is no clear y-value that your graph is approaching, or if there are two different y-values that your graph is approaching, then the limit does not exist (DNE).

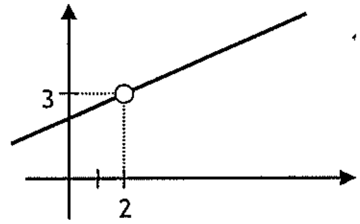
Example: Find the limit as x approaches 2 for each of the graphs below.

a.



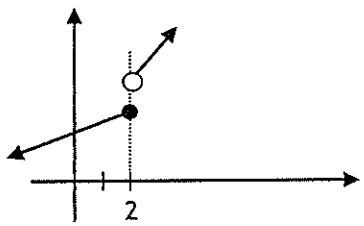
$\lim_{x\to 2} f(x) = 7$

b.



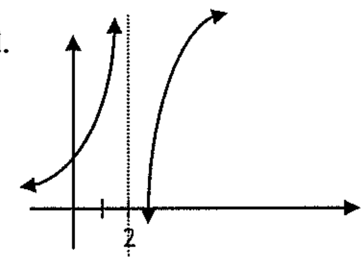
$\lim_{x\to 2} f(x) = 3$

c.



$\lim_{x\to 2} f(x)$ does not exist

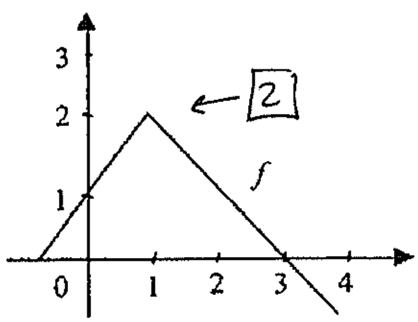
d.



$\lim_{x\to 2} f(x)$ does not exist

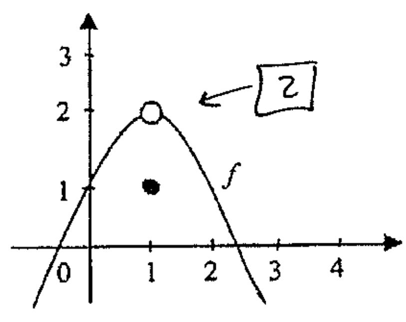
Your turn: The graphs of some functions are pictured below. Do you think that $\lim_{x\to 1} f(x)$ exists? If so, state its value.

1.



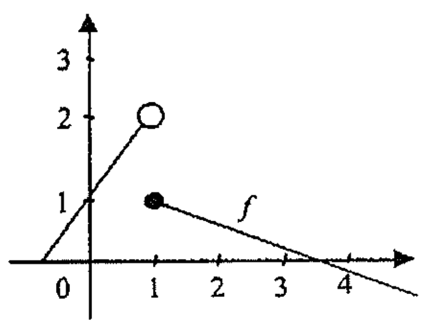
2

2.



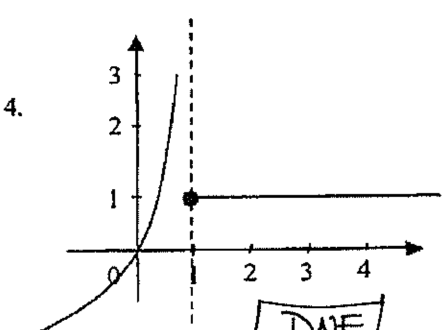
2

3.



DNE

4.



DNE

Solution: 1. 2; 2. 2; 3. DNE; 4. DNE