CALCULUS AB SUMMER ASSIGNMENT

Dear AP Calculus Students,

Welcome to AP Calculus. This is a rigorous, yet rewarding, math course. Most of the students who have taken Calculus in the past are amazed at how much they have to rely on prior knowledge from Algebra I and II and PreCalculus to complete a problem. Many times they find that it is not the Calculus steps that trip them up, but the embedded Algebra that needs to be done. To help you prepare for this course and make it through the "tedious algebra", I feel that it would be beneficial to show you some of the skills you will come across and have you practice them.

There are also many things that you have learned along the way that you need to have memorized by this point. Part II of this packet is a list of most of the things that you will need to have memorized to be successful in this course. Please don't memorize them for the moment and then forget. We will be using them ALL year long.

In addition to reviewing some of the past material learned, you need to get a jump start on the new material. In order to get everything taught in time, we would like to have you start to learn the first topic in Calculus – Limits. We have shown you examples and given you a few to try on your own. If you need further assistance, there are many web sites that have tutorials over Calculus material. One that we recommend is www.calculus-help.com. It has animated pictures of what is happening as you take a limit. You were also introduced to the topic in PreAP PreCal, although it may not have been called "limits". You should do this packet without a calculator unless indicated. The answers are given so that you may check you work.

In order to ensure that you complete the packet, we will be testing over this material sometime during the first two weeks of school. You will have a chance to get your questions answered the first week of school. Have fun this summer, but do a little bit of studying. A little studying now will save a lot of time this coming school year. We are going to have a great year in Calculus this fall. Get ready for an exciting, challenging, and rewarding school year.

Part 1 – Problems to Solve.

1. Simplify the following.

a. $\dfrac{5(x+h)^2-5x^2}{h}$

b. $\dfrac{(x-1)^2\,(3x-1)-2\,(x-1)\cdot 3}{(x-1)^4}$

c. $\dfrac{\frac{a}{b}-a}{a+\left(\frac{a}{b}\right)^2}$

d. $\dfrac{2x\,(x+1)^2-3\,(x+1)^3}{8x^3+14x^2+6x}$

e. $\dfrac{y(-1)-(-x)\left(-\frac{x}{y}\right)}{y^2}$

f. $\sqrt[3]{x^3+8}$

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2. Solve.

a. $\dfrac{3x+5}{(x-1)\,(x^4+7)}=0$

b. $(2x+1)(x-1)^2+(x+5)\,(2x+1)^2=0$

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3. Solve for y: $xy+xyz^2-3=5y+xz$

---

4. Write the expression as a sum of terms by splitting the fraction apart.

a. $\dfrac{u+1}{u}$

b. $\dfrac{u\sqrt{u}+\sqrt[3]{u}+1}{\sqrt{u}}$

c. $\dfrac{5x^3-2\sqrt{x}-4}{x^2}$

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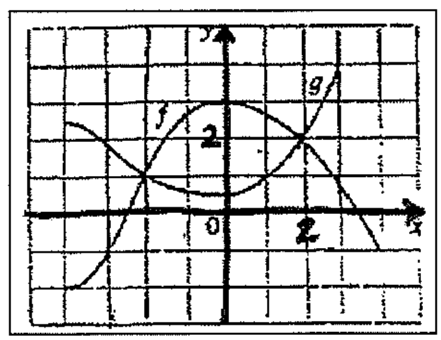
5. The graphs of f and g are given.



a. Find $f(-4)$ and $g(3)$.

b. For what values of x is $f(x)=g(x)$?

c. Estimate the solutions of the equation $f(x)=-1$.

d. On what interval(s) is f decreasing? Increasing?

e. State the domain and range of f.

f. State the domain and range of g.

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6. Given $f(x)=x-x^2$, find

a. $f(2+h)$

b. $f(x+h)$

c. $\dfrac{f(x+h)-f(x)}{h}$

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7. Given $f(x)=\dfrac{x}{x+1}$, find

a. $f(2+h)$

b. $f(x+h)$

c. $\dfrac{f(x+h)-f(x)}{h}$

---

8. If $f(x)=\frac{2}{3}x^{3/2}$, find $f(16)-f(4)$.

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9. What is the domain of each function?

a. $f(x)=\dfrac{x^4}{x^2+x-6}$

b. $f(x)=\sqrt{7-3x}$

c. $f(x)=\sqrt[3]{x-1}$

d. $f(x)=\ln\ x+3$

---

10. Find the domain and sketch a graph of each function.

a. $f(x)=\sqrt{4-x^2}$

b. $f(x)=x^2+2x-1$

c. $f(x)=|x|+x$ (calc.)

d. $f(x)=|2x|$

e. $f(x)=\dfrac{x}{|x|}$ (calc.)

f. $f(x)=\dfrac{x^2+5x+6}{x+2}$

g. $f(x)=\begin{cases} x & x\le 0 \\ x+1 & x>0 \end{cases}$

h. $f(x)=\begin{cases} 2x+3 & x<-1 \\ 3-x & x\ge -1 \end{cases}$

i. $f(x)=\begin{cases} \dfrac{x^2+3x+3}{x+1} & x\ne -1 \\ 3 & x=-1 \end{cases}$

j. $f(x)=\begin{cases} -1 & x\le -1 \\ 3x+2 & -1<x<1 \\ 7-2x & x\ge 1 \end{cases}$

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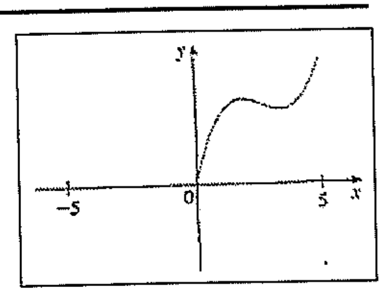
11. A function f has domain $-5,5$ and a portion of the graph is shown.

a. Complete the graph of f if it is known that f is an even function.

b. Complete the graph of f if it is known that f is an odd function.



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12. Describe the transformation of the parent function $y=f(x)$ for each of the following equations below.

a. $y=5f(x)$

b. $y=f(x-5)$

c. $y=-f(x)$

d. $y=f(5x)$

e. $y=f(x+5)-3$

f. $y=f(-x)$

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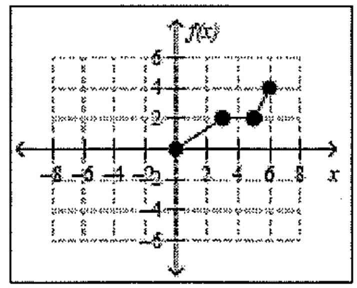
13. Given the graph of $y = f(x)$ below, graph the following transformations.



a. $f(2x)$

b. $f\left(\frac{1}{2}x\right)$

c. $f(-x)$

d. $|f(x)|$

---

14. Given $f(x) = \sqrt{x^2 - 16}$, find where $f(x) = 3$.

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15. Given $f(x) = \frac{4 - x^2}{x^2 + x}$, find where $f(x) = 0$.

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16. Given $f(x) = \sin\left(\frac{x}{2}\right) - 3$, find $f\left(\frac{\pi}{3}\right)$.

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17. Given $f(x) = \cos^2 3x - \sqrt{2}$, find $f\left(\frac{2\pi}{9}\right)$.

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18. Given $f(x) = \tan x + 1$, find where $f(x) = 0$.

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19. Given $f(x) = \sin^{-1} e^x$, find $f(0)$.

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20. Given $f(x) = \ln x - \cos^{-1}(x)$, find $f(1)$.

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21. Given $g(x) = -3f(x+1) - 2x + 3$ find $g(2)$.

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22. What is the equation of the line with slope $-\frac{2}{3}$ passing through the point $4, -1$.

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23. Find the equation of the line parallel to $2x + 3y = 4$ passing through $-1, 6$.

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24. Rewrite each absolute value function as a piece-wise function.

a. $f(x) = |x|$

b. $f(x) = |x + 3|$

c. $f(x) = |6 + 8x|$

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25. If $f(x)=x^2+8x-5$ and $g(x)=3x-1$, find $f\ \ g(2)\ \ -g\ \ f(2)$.

26. If $f(x)=x^2-4x+3$ and $g(x)=2x-1$, find $f\ \ g(x)$ and $g\ \ f(x)$.

27. Given $f(x)$ below, find $f^{-1}(x)$ (the inverse of $f(x)$).

a. $f(x)=4x-2$

b. $f(x)=e^{2x-2}$

c. $f(x)=\dfrac{x+2}{x-3}$

28. What is the vertex of the parabola $y=x^2+10x+20$? (do not use your calculator)

29. What are the asymptotes (both vertical and horizontal) for

a. $f(x)=\dfrac{2-x}{x^2-9}$

b. $f(x)=\dfrac{x-2}{x+3}$

c. $f(x)=\dfrac{x+3}{x^2-9}$

30. What is the slant asymptote for $f(x)=\dfrac{x^2-4x-6}{x+1}$?

31. An exponential curve is in the form of $y=Ce^{kx}$. If the curve has a y–intercept of 10 and passes through the point $5,5$, find the value of C and k.

32. Which of the following is NOT equal to $\sin x$?

a. $\dfrac{\cos x}{\cot x}$

b. $\csc x-\cot x\cos x$

c. $\dfrac{\sin 2x}{\sin x}$

d. all are

33. Which of the following is NOT equal to 1?

a. $\sin^2 x+\cos^2 x$

b. $\sin^3 x\cot^4 x\sec x$

c. $\tan x-\sec x\ \ \tan x+\sec x$

34. Solve: $2\cos x-1=0$

35. Solve: $2\sin^2 x+\sin x=1$

36. Given the tables of values below for $f(x)$ and $g(x)$, find

x	$f(x)$
1	4
2	3
3	5
4	2
5	1

x	$g(x)$
1	3
2	5
3	2
4	1
5	4

a. $g\ f(3)$

b. $g^{-1}\ f^{-1}(1)$

---

37. Given the function $h(x)=\begin{cases} x^2-4x+3 & x<3 \\ 2x-9 & x\geq 3 \end{cases}$ find

a. $h(0)$

b. $h(3)$

c. $h(5)$

d. the minimum value of $h(x)$

e. when $h(x)=8$

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38. A swimming pool can hold a maximum of 360 gallons of water. The full pool develops a leak and is losing water at a constant rate. After 3 hours the pool has 354 gallons of water in it.

a. Write a function $g(t)$ for the total number of gallons of water that is in the pool in terms of the time, t, the number of hours since the pool developed the leak.

b. Find $g(20)$. Explain the meaning of the answer in the context of the problem.

c. If the leak is fixed after 20 hours and the owner immediately begins to fill the pool at a rate of 4 gallons of water per hour, how long will it be before the pool is full again?

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39. Given $g(x)=\begin{cases} \frac{x}{2}, & \text{if } x\geq 4 \\ \sqrt{x}, & \text{if } 0<x<4 \\ x^2, & \text{if } x<0 \end{cases}$, find

a. $g(-3)$

b. $g(1)$

c. $g(0)$

d. Is $g(x)$ continuous?

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40. Evaluate the following.

a. $\log_2 16$

b. $\log_3 1$

c. $\log 10$

d. $\ln 1$

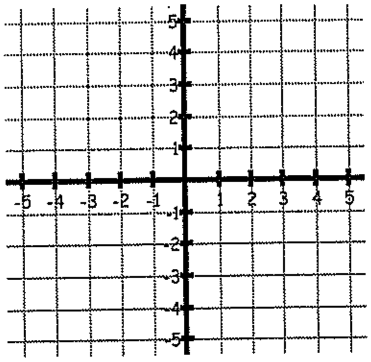
e. $\ln e$

f. $\ln e^3$

Part 2 – Things that need to be memorized!

Graphs of Functions

$f(x) = x^2$

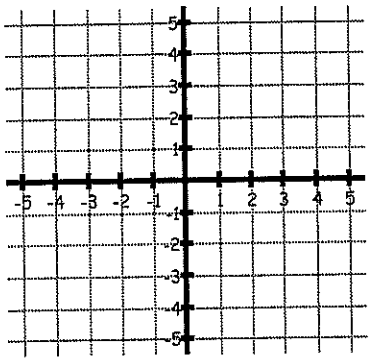


Max: ________ Min: ________

Domain: ________ Range: ________

$\lim_{x\to\infty} f(x) =$ _____ $\lim_{x\to-\infty} f(x) =$ _____

$f(x) = x^3$

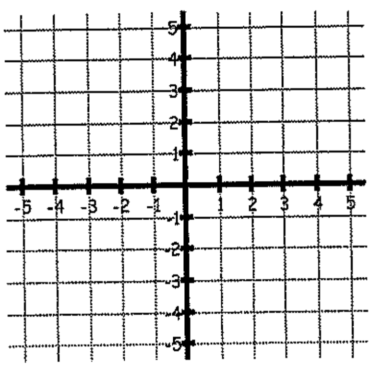


Max: ________ Min: ________

Domain: ________ Range: ________

$\lim_{x\to\infty} f(x) =$ _____ $\lim_{x\to-\infty} f(x) =$ _____

$f(x) = |x|$

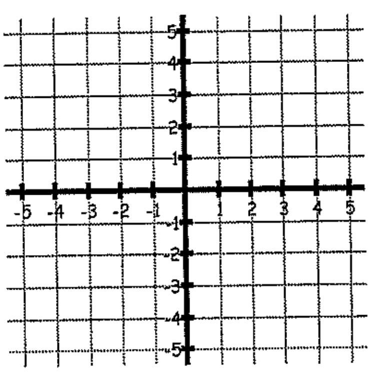


Max: ________ Min: ________

Domain: ________ Range: ________

$\lim_{x\to\infty} f(x) =$ _____ $\lim_{x\to-\infty} f(x) =$ _____

$f(x) = \sin x$

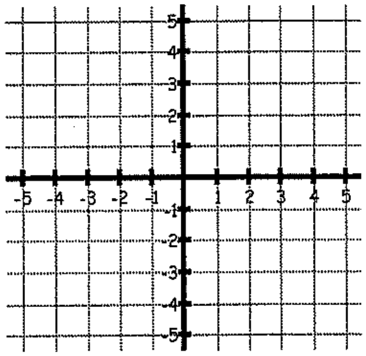


Max: ________ Min: ________

Domain: ________ Range: ________

Period: ________

$f(x) = \cos x$

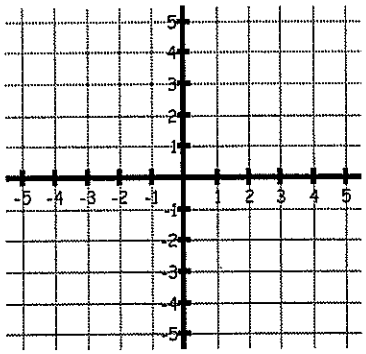


Max: ________ Min: ________

Domain: ________ Range: ________

Period: ________

$f(x) = \tan x$



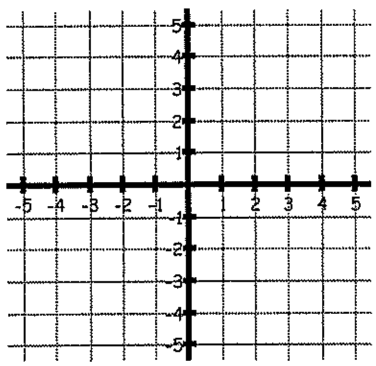
Max: ________ Min: ________

Domain: ________ Range: ________

Period: ________

$\lim_{x \to \pi/2^-} f(x) =$ ______ $\lim_{x \to \pi/2^+} f(x) =$ ______

$f(x) = \tan^{-1} x$

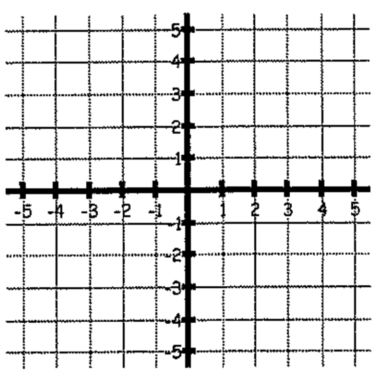


Domain: ________ Range: ________

Hor. Asymptote(s): ________

$\lim_{x \to \infty} f(x) =$ ______ $\lim_{x \to -\infty} f(x) =$ ______

$f(x) = \sqrt{x}$

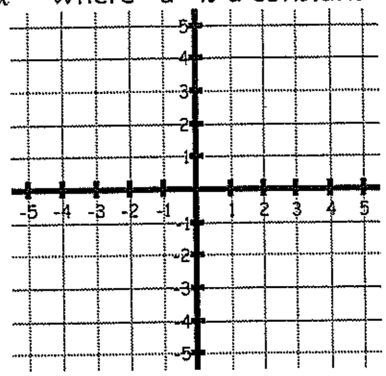


Max: ________ Min: ________

Domain: ________ Range: ________

$\lim_{x \to \infty} f(x) =$ ______ $\lim_{x \to 0^+} f(x) =$ ______

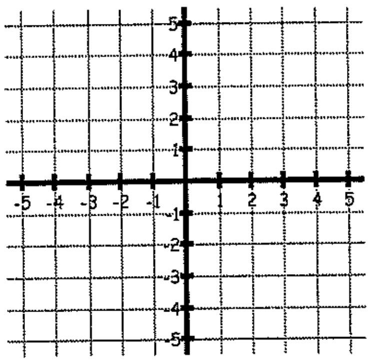
$f(x)=\sqrt{a^2-x^2}$ where "a" is a constant



Max: ________ Min: ________

Domain: ________ Range: ________

$f(x)=e^x$



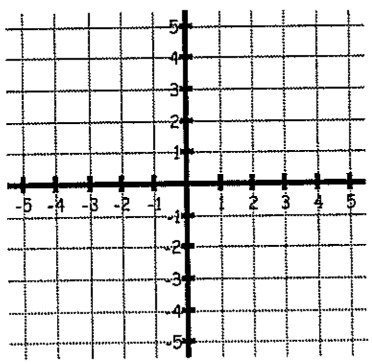
Max: ________ Min: ________

Domain: ________ Range: ________

Hor. Asymptote: ________

$\lim_{x\to\infty} f(x)=$ ______ $\lim_{x\to-\infty} f(x)=$ ______

$f(x)=e^{-x}$



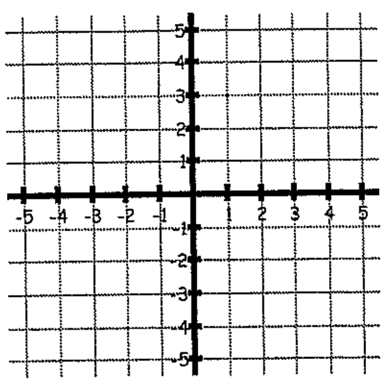
Max: ________ Min: ________

Domain: ________ Range: ________

Hor. Asymptote: ________

$\lim_{x\to\infty} f(x)=$ ______ $\lim_{x\to-\infty} f(x)=$ ______

$f(x)=\ln x$



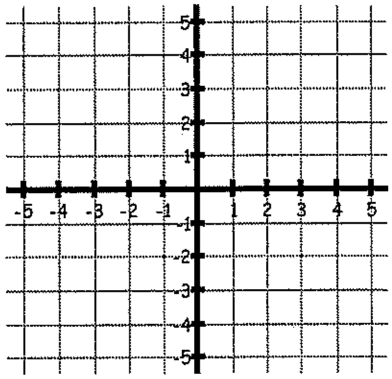
Max: ________ Min: ________

Domain: ________ Range: ________

Vert. Asymptote: ________

$\lim_{x\to\infty} f(x)=$ ______ $\lim_{x\to0^+} f(x)=$ ______

$f(x) = \frac{1}{x}$



Max: ________ Min: ________

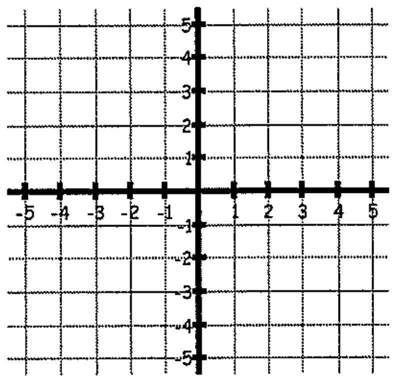
Domain: ________ Range: ________

Hor. Asym: ______ Vert. Asym: ____

$\lim_{x \to -\infty} f(x) =$ ___ $\lim_{x \to \infty} f(x) =$ ___

$\lim_{x \to 0^+} f(x) =$ ___ $\lim_{x \to 0^-} f(x) =$ ___

$f(x) = \frac{1}{x^2}$



Max: ________ Min: ________

Domain: ________ Range: ________

Hor. Asym: ______ Vert. Asym: ____

$\lim_{x \to -\infty} f(x) =$ ___ $\lim_{x \to \infty} f(x) =$ ___

$\lim_{x \to 0^+} f(x) =$ ___ $\lim_{x \to 0^-} f(x) =$ ___

Exponential and Logarithmic Properties

$\ln 1 =$ ______ $\ln e =$ ______ $\ln e^2 =$ ______ $\ln e^a =$ ______

$\ln ab =$ ________________ $\ln\left(\frac{a}{b}\right) =$ ________________

$\ln a^b =$ ____________

Trig Values

$\sin 0 =$ ______ $\cos 0 =$ ______ $\tan 0 =$ ______

$\sin\left(\frac{\pi}{6}\right) =$ ______ $\cos\left(\frac{\pi}{6}\right) =$ ______ $\tan\left(\frac{\pi}{6}\right) =$ ______

$\sin\left(\frac{\pi}{4}\right) =$ ______ $\cos\left(\frac{\pi}{4}\right) =$ ______ $\tan\left(\frac{\pi}{4}\right) =$ ______

$\sin\left(\frac{\pi}{3}\right) =$ ______ $\cos\left(\frac{\pi}{3}\right) =$ ______ $\tan\left(\frac{\pi}{3}\right) =$ ______

$\sin\left(\frac{\pi}{2}\right) =$ ______ $\cos\left(\frac{\pi}{2}\right) =$ ______ $\tan\left(\frac{\pi}{2}\right) =$ ______

$\sin\left(\frac{5\pi}{6}\right) =$ ______ $\cos\left(\frac{5\pi}{6}\right) =$ ______ $\tan\left(\frac{5\pi}{6}\right) =$ ______

$\sin\left(\frac{3\pi}{4}\right) =$ ______ $\cos\left(\frac{3\pi}{4}\right) =$ ______ $\tan\left(\frac{3\pi}{4}\right) =$ ______

$\sin\left(\frac{2\pi}{3}\right) =$ ______ $\cos\left(\frac{2\pi}{3}\right) =$ ______ $\tan\left(\frac{2\pi}{3}\right) =$ ______

$\sin\ \pi =$ ______ $\cos\ \pi =$ ______ $\tan\ \pi =$ ______

$\sin\left(\frac{7\pi}{6}\right) =$ ______ $\cos\left(\frac{7\pi}{6}\right) =$ ______ $\tan\left(\frac{7\pi}{6}\right) =$ ______

$\sin\left(\frac{5\pi}{4}\right) =$ ______ $\cos\left(\frac{5\pi}{4}\right) =$ ______ $\tan\left(\frac{5\pi}{4}\right) =$ ______

$\sin\left(\frac{4\pi}{3}\right) =$ ______ $\cos\left(\frac{4\pi}{3}\right) =$ ______ $\tan\left(\frac{4\pi}{3}\right) =$ ______

$\sin\left(\frac{3\pi}{2}\right) =$ ______ $\cos\left(\frac{3\pi}{2}\right) =$ ______ $\tan\left(\frac{3\pi}{2}\right) =$ ______

$\sin\left(\frac{11\pi}{6}\right) =$ ______ $\cos\left(\frac{11\pi}{6}\right) =$ ______ $\tan\left(\frac{11\pi}{6}\right) =$ ______

$\sin\left(\frac{7\pi}{4}\right) =$ ______ $\cos\left(\frac{7\pi}{4}\right) =$ ______ $\tan\left(\frac{7\pi}{4}\right) =$ ______

$\sin\left(\frac{5\pi}{3}\right) =$ ______ $\cos\left(\frac{5\pi}{3}\right) =$ ______ $\tan\left(\frac{5\pi}{3}\right) =$ ______

$\sin x$ is positive in Quadrants ______________

$\cos x$ is positive in Quadrants ______________

$\tan x$ is positive in Quadrants ______________

Trig Identities

Pythagorean Identities:

$\sin^2 x + \cos^2 x =$ ______ $1 + \cot^2 x =$ ________ $\tan^2 x + 1 =$ ______

Double Angle Identities:

$\sin 2x =$ ______________

$\cos 2x =$ ________________ = ________________ = ________________

Reciprocal Identities:

$\csc x =$ $\sec x =$ $\cot x =$

Inverse Trig Values

$\sin^{-1}\left(\frac{1}{2}\right) =$ _______ $\cos^{-1}\left(\frac{1}{2}\right) =$ _______ $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) =$ _______

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) =$ _______ $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) =$ _______ $\tan^{-1} 1 =$ _______

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ _______ $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) =$ _______ $\tan^{-1} \sqrt{3} =$ _______

$\sin^{-1} 0 =$ _______ $\cos^{-1} 0 =$ _______ $\tan^{-1} 0 =$ _______

$\sin^{-1} 1 =$ _______ $\cos^{-1} 1 =$ _______ $\tan^{-1} \infty =$ _______

$\sin^{-1}\left(-\frac{1}{2}\right) =$ _______ $\cos^{-1}\left(-\frac{1}{2}\right) =$ _______ $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) =$ _______

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ _______ $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ _______ $\tan^{-1} -1 =$ _______

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ _______ $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ _______ $\tan^{-1} -\sqrt{3} =$ _______

$\sin^{-1} -1 =$ _______ $\cos^{-1} -1 =$ _______ $\tan^{-1} -\infty =$ _______

Part 3: An Introduction to Limits

The limit of a function is the y-value that you are getting close to as x gets close to some number in the domain. We write $\lim_{x \to a} f(x)$, which is read "the limit of $f(x)$ as x approaches a". The limit must be the same as x approaches "a" on both the left and the right.

There are many ways to find a limit. We will focus on three main ways for now: from a graph, from a table, and by direct substitution. Study the following examples of each type and then try the sample exercises on your own.

To find the limit from a table, look at the y-values as the x values get closer and closer to your "a" value. See if there is one number that all y-values seem to be going towards.

Example: Find $\lim_{x \to 2} x^2$.

X	1.9	1.99	1.999	2.001	2.01	2.1
Y	3.61	3.9601	3.996	4.004	4.0401	4.41

→ 4 ←

Solution: We are looking for the y-value as our x-values get closer and closer to 2. Looking at the chart from both the left and right sides, we can see that our y-values are getting closer and closer to 4. Therefore, $\lim_{x \to 2} x^2 = 4$.

Your turn: Find the following limits using the charts.

1. $\lim_{x \to 3} x^2 - 1$.

X	2.9	2.99	2.999	3.001	3.01	3.1
Y	7.41	7.9401	7.994	8.006	8.0601	8.61

2. $\lim_{x \to -1} 2x$.

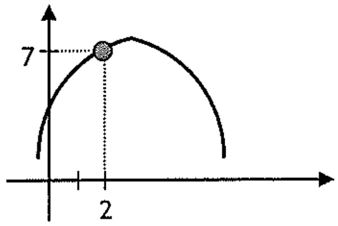
X	-0.9	-0.99	-0.999	-1.001	-1.01	-1.1
Y						

Solution: 1. 8; 2. -2

To find a limit from a graph, we follow the graph on either side of our "a" value towards that "a" value. The answer will be whatever y-value our graph is approaching. It is important to note that the graph does not have to actually "hit" that y-value. A limit is simply "what the y-value should be." If there is no clear y-value that your graph is approaching, or if there are two different y-values that your graph is approaching, then the limit does not exist (DNE).

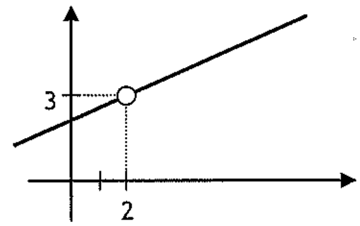
Example: Find the limit as x approaches 2 for each of the graphs below.

a.



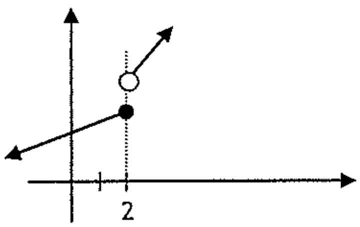
$\lim_{x\to 2} f(x) = 7$

b.



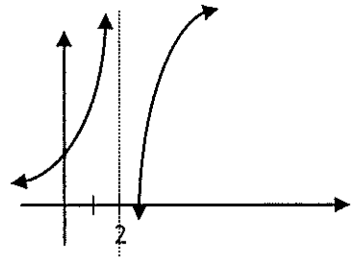
$\lim_{x\to 2} f(x) = 3$

c.



$\lim_{x\to 2} f(x)$ does not exist

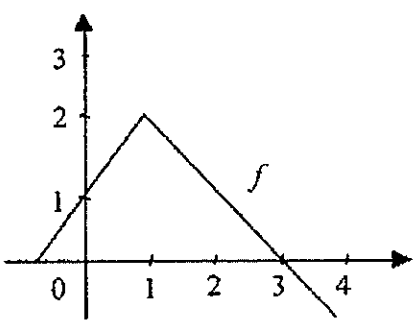
d.



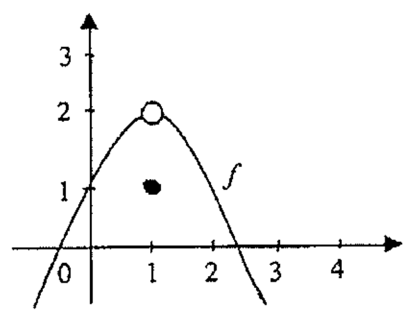
$\lim_{x\to 2} f(x)$ does not exist

Your turn: The graphs of some functions are pictured below. Do you think that $\lim_{x\to 1} f(x)$ exists? If so, state its value.

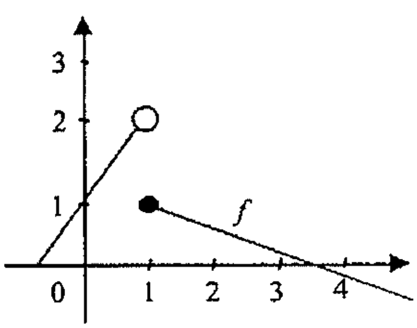
1.



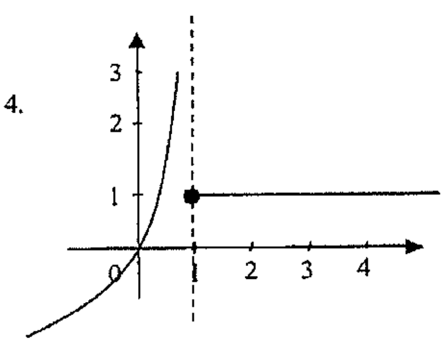
2.



3.



4.



Solution: 1. 2; 2. 2; 3. DNE; 4. DNE